RESEARCH ARTICLE

NUMERICAL STUDY OF THE BEHAVIOR OF A PILE SUBJECTED TO AXIAL AND LATERAL LOADS AND COMPARISON WITH THE ANALYTICAL APPROACH

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ABSTRACT

This study explores the modeling of the behavior of piles used as deep foundations when subjected to axial or lateral loads and highlights the importance of taking soil-pile interaction into account. M Numerical modeling was performed using *Robot Structural Analysis (& REX Piles)* software, recognized for structural calculations, allowing an alternative numerical approach to analytical resolution. *REX Piles* is a complement to *Robot Structural Analysis (RSA)* that allows the elastoplastic interaction between piles and soil to be taken into account in the analysis of structures resting on piles. It allows the elastic characteristics of the pile base support to be specified and the distribution of reaction modules (horizontal and vertical) along the pile to be calculated. Subsequently, after applying the loads, the results of the internal forces and displacements along the pile were assessed using *RSA*. This comparative study reveals a similarity in the diagrams of the internal forces transmitted. The lateral displacements are almost identical, which can be explained by the use of the same calculation model, the Winkler model, in both methods. However, the values of the vertical displacements along the pile differ due to the distinct calculation approaches of the two methods.

INTRODUCTION

A pile is a type of deep foundation designed to transfer loads from a structure to soil layers located at depths ranging from a few meters to several tens of meters, when the surface soils do not have sufficient strength to be used with shallow foundations. However, understanding the behavior of these pile-type foundations when subjected to axial or lateral loads is essential for designing safe and durable foundations. It is in this context that the present study focuses on the numerical modeling of the behavior of piles subjected to axial or lateral loads using *Robot Structural Analysis* software and its *REX Piles* add-on module, in comparison with a purely analytical approach, while highlighting the importance of soil-foundation interaction. For both the axial load and lateral load studies, we will present the analytical solution method used to determine the expressions of internal forces and displacements as a function of depth. Using *REX-Piles*, we will establish the evolution curves of the reaction coefficients. Finally, we will perform a comparative study of the curves showing the evolution of internal forces and displacements (axial and lateral) obtained with *Robot Structural Analysis* and those resulting from the analytical approach in order to evaluate and discuss the results.

1. PILE UNDER AXIAL LOAD

1.1. Analytical Calculation Procedure

Various methods for evaluating the settlement of a pile under axial loading are presented in the literature. These include empirical methods (Meyerhof, 1956; Frank, 1995; Vesic, 1977; etc…), the elasticity theory method (Poulos, 1968; Randolph & Wroth, 1978; Banerjee and Butterfield, 1978; etc...), the load transfer theory method (Coyle and Reese, 1966; Hirayama, 1990; Fleming, 1992; etc.) and numerical methods based on finite element or finite difference calculations (*Plaxis, Flac, REX-Pile,* etc.). The most suitable method for a purely analytical calculation is that of load transfer curves, which can be used to obtain expressions for the axial force and settlement along the pile. It is based on the progressive mobilization of axial friction on the pile shaft τ or the stress under the pile base q_p with the relative soil-pile displacement s (Figure 1). This method hasbeen widely used to estimate the displacement of a pile under axial loading, and its results are considered satisfactory (Frank and Zhao, 1982; Maleki, 1995). According to this method, soil/deep foundation interaction is represented by assimilating the soil around the foundation to a series of nonlinear springs, behaving independently of each other.
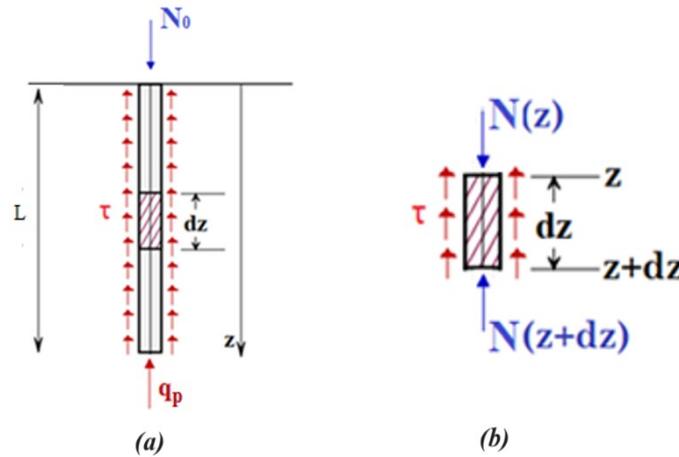
Figure 1. - (a) Pile under axial load N_0, (b) Pile section under normal stress N

In the following, let us denote by:

$E_p A_p$: the compressive stiffness of the pile
P: perimeter of the pile
L: length of the pile

The differential equation of equilibrium for a beam under compression or tension (Frank 1989).

$$E_p A_p \frac{d^2 s}{dz^2} - P.\tau(s) = 0 \qquad (1)$$

Several authors (Verbrugge, 1981; Frank and Zhao, 1982; Hirayama, 1992; Abchir and Burlon, 2016) have proposed expressions to describe the evolution of axial friction $\tau(s)$ mobilized as a function of settlement s at a point and tip resistance as a function of tip settlement $q_p(s_p)$. Among these models, we have chosen that of Frank and Zhao, as it has proven to be effective and accurate in calculating settlements and has been adopted by the French regulations on the design and calculation of foundations and civil engineering structures, Fascicule 62, Title V (1993). This model also allows for an analytical solution of the differential equation and better use of the results. The Frank and Zhao (1982) model for the mobilization curves of axial friction $\tau(s_z)$ and peak load $q(s_p)$ is trilinear and is based on pressuremeter results (Figure 2). The stiffness of the soil-pile interface is a function of the soil's pressure module E_M, the pile diameter D, and the nature of the soil (fine or granular). The initial slopes k_τ and k_q are given respectively for fine soils (clays and silts) and granular soils. As this study concerns clay soils, we will focus below on the values of k_τ and k_q for fine soils ($k_\tau = 2E_M/D$ and $k_q = 11E_M/D$). Let E_M be the pressiometric modulus at the depth considered and D the diameter of the pile.
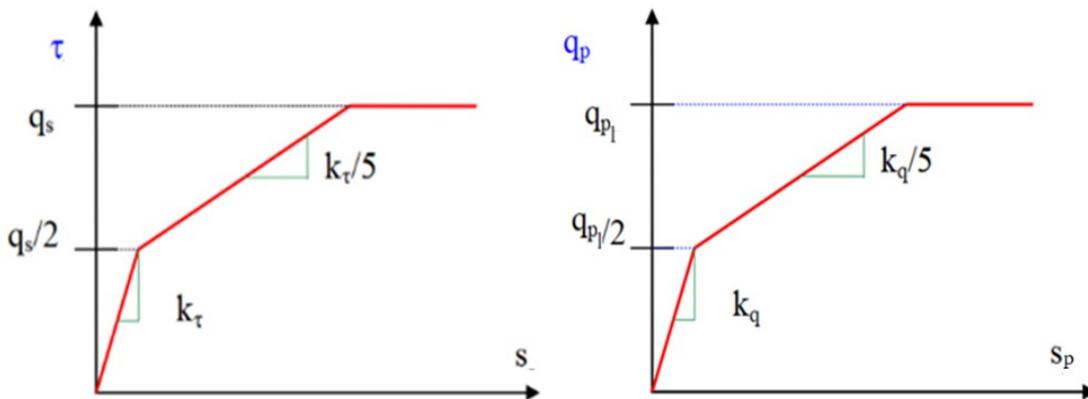

Figure 2. Law of mobilization of lateral friction and unit tip resistance (Frank and Zhao, 1982)

Solving the differential equation allows us to find the expression for vertical displacement (settlement) $s(z)$ and we can derive that of the normal force $N(z)$ at any position z of the pile with the equation

$$N(z) = -EA \frac{ds}{dz}. \qquad (2)$$

These expressions will enable us to plot the curves showing the evolution of the normal stress and vertical displacement along the pile.

1.2. Numerical Modeling With *Robot Structural Analysis*

Modeling using *Robot Structural Analysis* (*RSA*) software with its **REX Piles** add-on module will enable a comparative analysis of the results with those of the analytical calculation, both for verification purposes and to identify the differences, similarities, and specificities of each approach. The objective is to deepen our understanding of the behavior of piles under axial loading.

REX-Piles is an add-on to *Robot Structural Analysis* that allows the elastoplastic interaction between piles and soil to be taken into account in the analysis of structures resting on piles. It allows the elastic characteristics of the pile base support to be specified and the reaction modulus distributions (horizontal and vertical) along the pile to be calculated for all the springs distributed along the pile axis (Figure 3). Subsequently, after applying the loads, the results relating to the normal force and displacement- long the pile can be assessed on *RSA*. Table 1 gives the values of the soil and pile input parameters on *REX Piles*.

Table 1. Input data on REX-Piles

D (m)	γ_s (kN/m³)	γ_d (kN/m³)	E_o (MPa)	q_0 (MPa)	t_0 (MPa)	ν
0.8	27	20	3.73	0.2	0.02	0.3

With:

γ_s : Specific weight of solid soil carcass

γ_d : Density of solid soil carcass

E_o : Oedometer modulus of the soil

q_0 : Unit resistance limit of the soil under the base of the pile, equivalent to q_{pl}

t_0 : Unit resistance limit of the soil along the lateral surface of the pile, equivalent to q_s

ν: Poisson's ratio of the soil

REX-Piles is an *RSA* add-on module that integrates soil-pile interaction. It has a built-in soil database that allows you to define soil layers with their geotechnical parameters (Figure 3). After entering the pile parameters, the user chooses a soil model, assigns the characteristics of the different layers, and selects a soil-pile interaction model (e.g., the elastic support model).
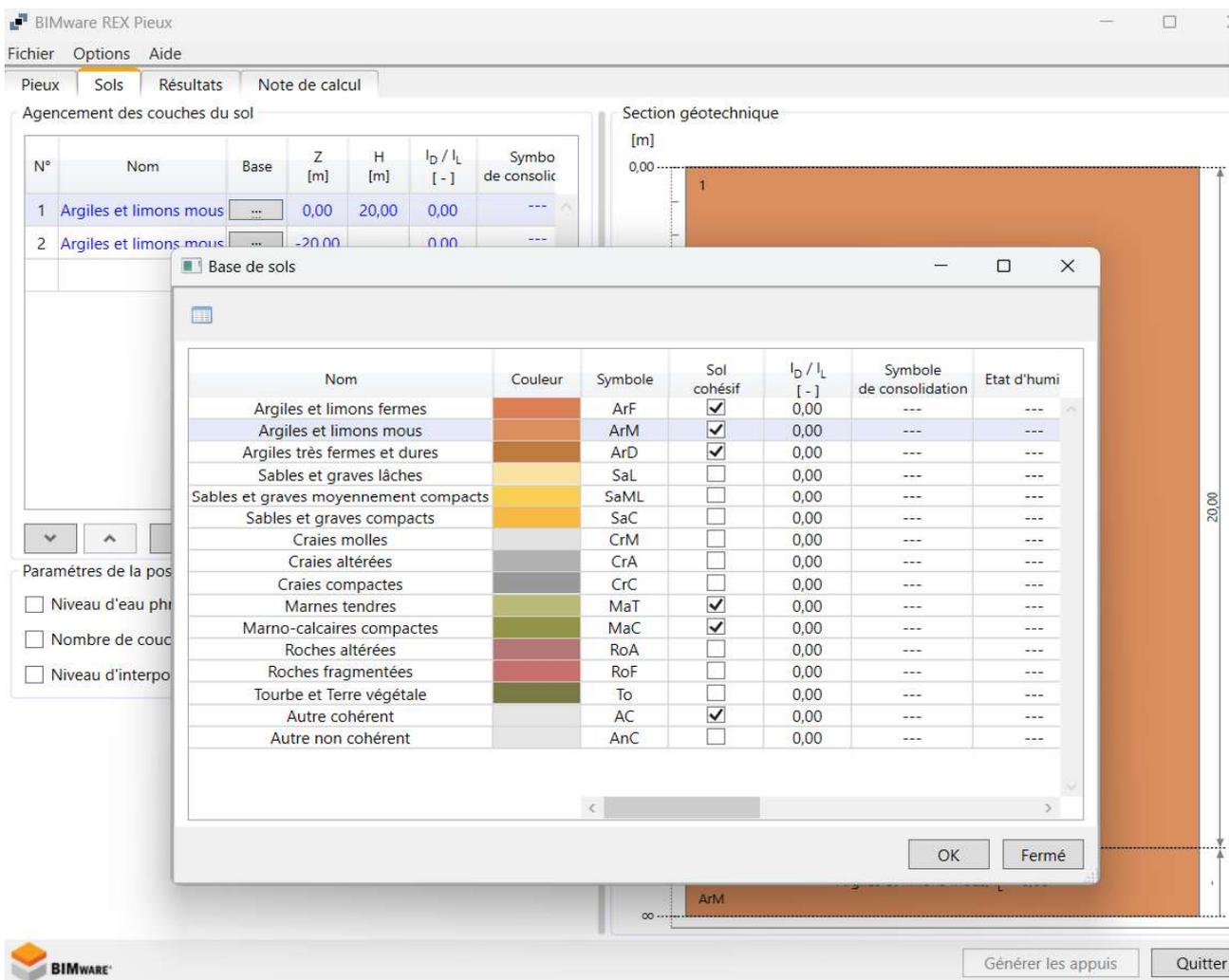

Figure 3. Definition of soil layers in REX-Piles

After defining the pile and soil, *REX-Piles* calculates the stiffness coefficients. Figure 4 shows the curve of the vertical reaction coefficient as a function of pile depth.

These reaction coefficient values obtained along the pile using REX-Piles will be used to generate the supports assigned their elastic characteristics. Figure 5 below shows the discretization and generation of intermediate elastic supports for the pile using the software (*RSA*).
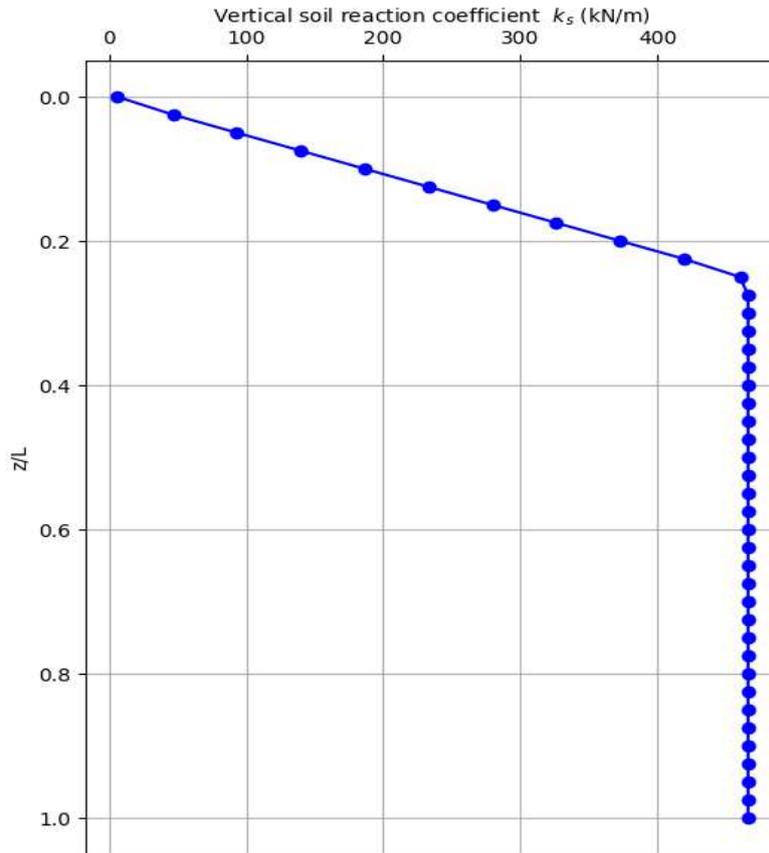
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with the analytical approach

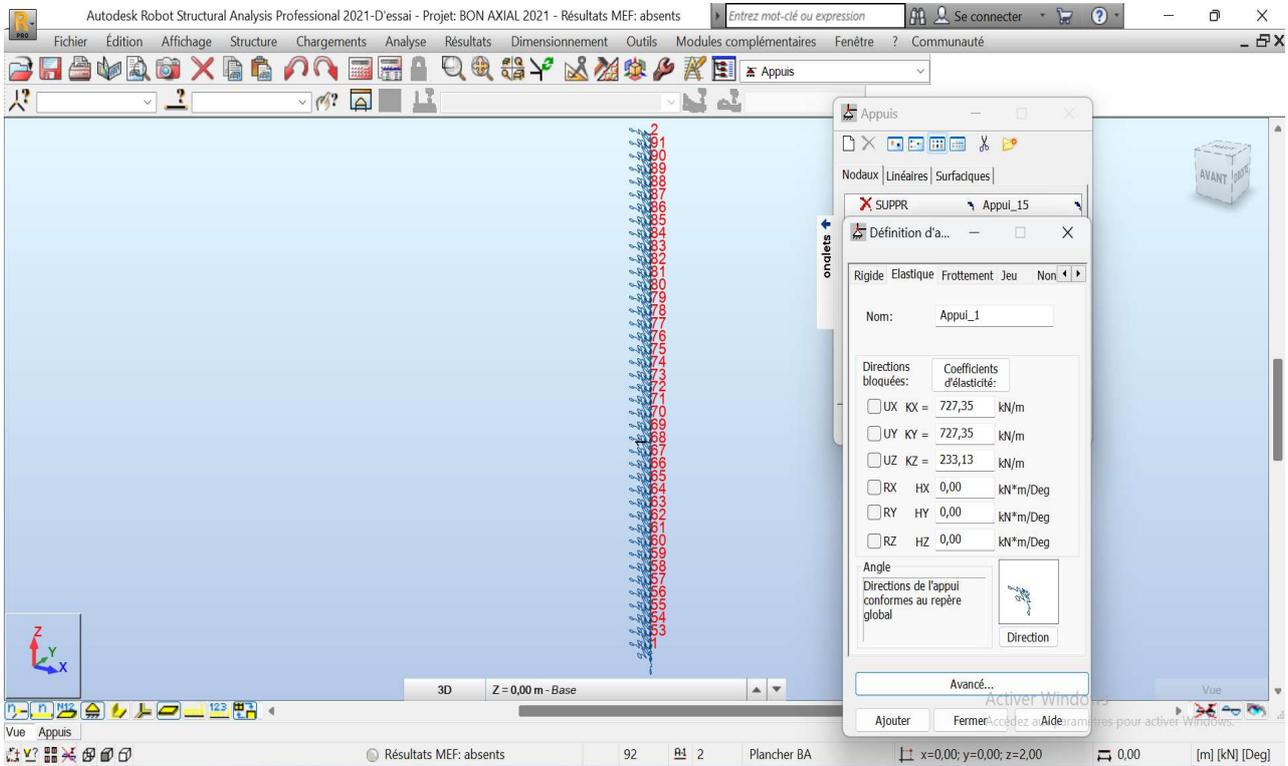**Figure 4. - Evolution of the vertical reaction coefficient of the soil (REX-Piles)**

Figure 5. Discretization and generation of elastic supports along the pile in Robot Structural Analysis

Figure 6 shows the comparative study of the normal force N as a function of the normalized depth z/L for a pile, obtained using two different methods: the finite element method *(RSA)* and the analytical method (based on the Frank and Zhao model). The results obtained using the two approaches show a decrease in normal stress N with relative depth z/L, which is consistent with the physics of the problem. There is also a convergence of stress values at the pile head and tip. The value of the load at the pile head used is N_0 equal to 0.4 MN.
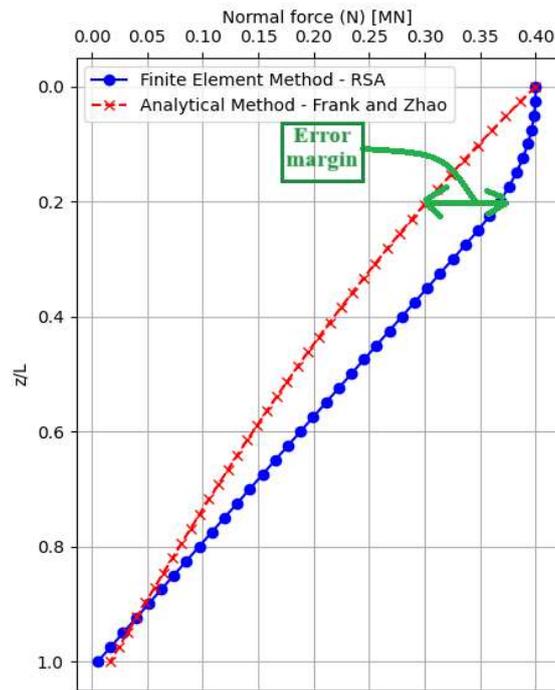
Figure 6. Comparison of the evolution of normal stress (Analytical & Numerical)

The analytical curve (Frank and Zhao) has a slightly steeper slope than the curve obtained using the RSA method, which shows faster stress dissipation in the upper half of the pile. The differences observed can be attributed to the underlying assumptions of the two methods. The *Robot Structural Analysis &REX-Piles* method, using finite elements, can mobilize local effects and nonlinearities that the analytical method simplifies or ignores. REX-Piles calculates the variability of reaction modules along the pile. It can therefore be seen that its pile calculation model more fully represents the actual behavior of the pile in the soil. The pile-soil interaction is represented in the model by a system of elastic supports distributed along the axis of the element and elastic supports at the base of the pile. The finite element method can provide a more accurate solution in terms of spatial discretization, while the analytical method relies on closed solutions that may not take into account all the complex aspects of soil and pile behavior. The Frank and Zhao method, being trilinear, is very well suited for calculating piles under axial load, but has certain limitations, such as the need to proceed by successive iterations to determine the final values of A_0 and B_0, the complexity of its application in the presence of stratified and heterogeneous soils, and the variations induced by the compressibility of the pile. The two evolution curves offer valuable insights into the study of normal stress distribution in a pile. The finite element method using software (*RSA*) provides a potentially more accurate numerical solution for specific conditions and complex geometries, while the analytical method (Frank and Zhao) offers an effective and valid solution for calculating the pile, taking into account creep load compliance. The maximum margin of error when comparing these two evolution curves is 23.33%, obtained at z = 0.2 L. The differences between the two methods must be carefully considered in the design and analysis of deep foundations, taking into account the limitations and assumptions of each approach.

The evolution of axial displacement (settlement) along the pile obtained by *RSA* modeling is shown in Figure 7.
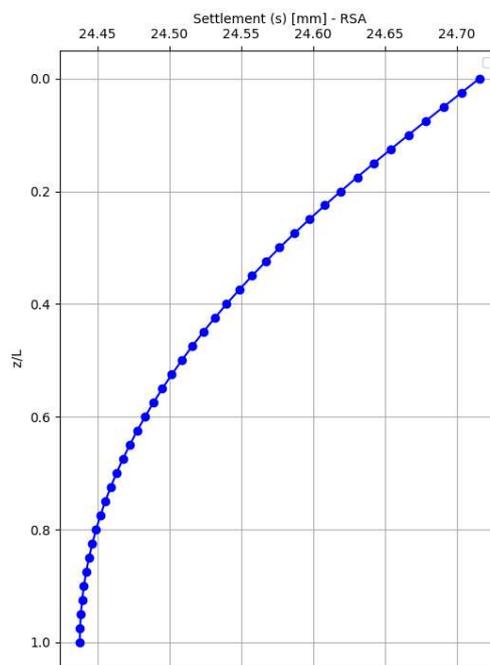

Figure 7. Evolution of settlement as a function of depth based on RSA

REX-Piles analyzes soil-pile interaction by taking into account the variability of the vertical reaction modulus along the pile shaft. Figure 7 shows a gradual decrease in settlement *s* with normalized depth *z/L*, illustrating the effect of the load applied at the pile head and its propagation along the shaft. Settlement is always maximum at the head (s(0))and minimum at the tip of the pile, and the decrease is always hyperbolic; the same observation was made in the analytical study of settlement using the Frank and Zhao method. However, the settlement values according to this numerical method are not comparable to the values obtained by the analytical study, which clearly shows that these methods do not follow the same approach. The Frank and Zhao model is a method based purely on the mobilization of lateral friction along the pile (general principle of *t-z* load transfer curve methods*)* for calculating settlement and axial force transmitted by limiting the value of the applied load in relation to the soil's creep load. Having chosen a layer of soft clay soil with a fairly low pressiometric modulus in this study, we end up with a relatively low creep load, hence the low settlement values obtained by this method. The REX-Piles numerical method is based more on determining the values of the longitudinal reaction modulus along the pile. The reaction modulus method is a commonly used method based on Winkler's general method, a model available at on *RSA* and *REX-Piles*. The relationships giving the calculation expression for the reaction coefficient vary from one author to another. In addition, it should be noted that the approaches of Frank and Zhao and that of *RSA* are different, which may explain certain discrepancies in some results.

2   PILE UNDER LATERAL LOAD

2.2   ANALYTICAL CALCULATION PROCEDURE

When a foundation is subjected to lateral forces at its head, it will move to mobilize reactions (*p*) in the soil, thus balancing the moment $M_0$and/or the lateral force V_0 (Figure 8).
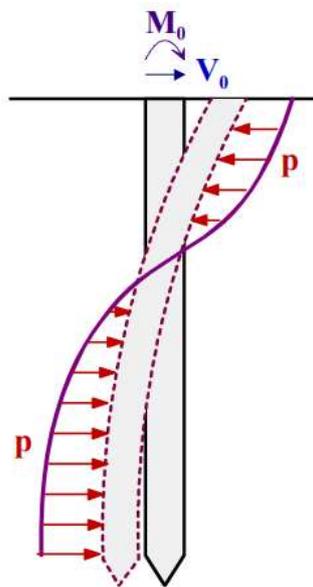

Figure 8. Mobilization of lateral soil reaction by a pile (Plumelle, 2005)

The behavior of the foundation depends both on its own bending stiffness *(E_p, I_p)* and that of the soil (reaction modulus *E_s*), i.e., on the relative pile-soil stiffness. Consider a flexible pile subjected to a moment $M_0$and a force V_0at its head in linearly elastic clay (Figure 9).
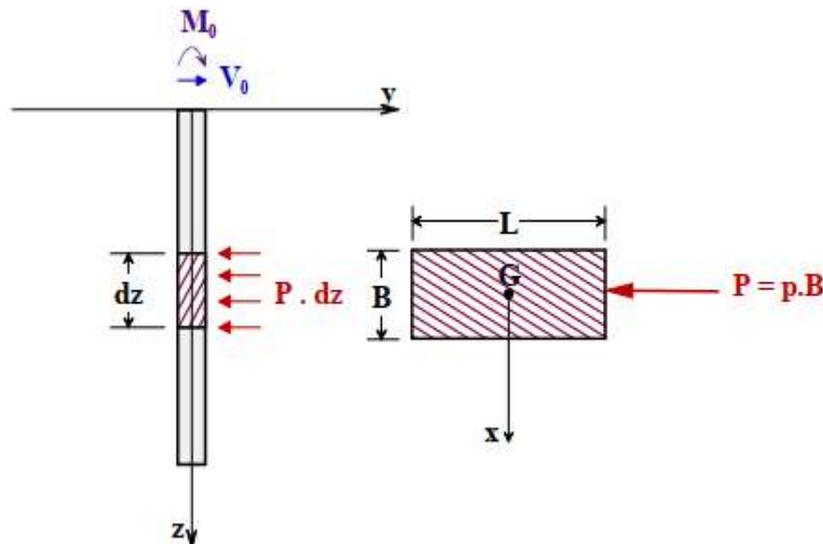

Figure 9. Diagram of a pile under M_0 and V_0 (Plumelle, 2005)

By isolating a section of beam loaded by a distributed load *P* and delimited by two infinitely close cross sections separated by *dz*, we obtain the equations for straight beams subjected to a uniformly distributed force P (soil reaction along the pile):

$$\begin{cases} P = -\dfrac{dV}{dz} \\ V = \dfrac{dM}{dz} \\ M = E_P I_P \dfrac{d^2 y}{dz^2} \end{cases} \tag{3}$$

Where V and M , the shear force and bending moment at depth z.

The soil reaction law is considered to be linear elastic of the Winkler type (1867):

$$P = E_s \cdot y \tag{4}$$

Several authors (Terzaghi, 1955; Ménard et al., 1969; Poulos, 1971; Bowles, 1977) have proposed a formulation of the soil reaction modulus E_s. In this study, we adopt the model of *Ménard et al.* This choice is explained by the fact that it takes into account many more parameters (*Eq. 5*), thus allowing for a better characterization of the interaction between soil and pile.

$$E_s = \begin{cases} \dfrac{3E_M}{\frac{2}{3}\left(\frac{D_0}{D}\right)\left(2{,}65\frac{D}{D_0}\right)^\alpha + \frac{\alpha}{2}} & \text{pour } D > D_0 \\ \dfrac{18\ E_M}{4(2{,}65)^\alpha + 3\alpha} & \text{pour } D < D_0 \end{cases} \tag{5}$$

D_0: Reference diameter equal to 0.6
α : Rheological coefficient depending on soil type
E_M : Soil pressure module

From equations (*3*) *and (4)*, we can deduce the fourth-order linear differential equation for the deformation of the pile:

$$E_p I_p \frac{d^4 y}{dz^4} + E_s \cdot y = 0 \tag{6}$$

Solving this differential equation allows us to find the expressions for lateral displacement $y(z)$, and using equations *(3)* we can derive the expressions for shear force $V(z)$, and bending moment $M(z)$, at any position z, of the pile. From these expressions, we can assess the curves showing the evolution of lateral displacement, shear force, and bending moment along the pile. Note that, for a laterally loaded pile, a distinction is made between rigid and flexible piles. If the pile base $L > 3l_0$, the pile is said to be flexible. If $L < l_0$, it becomes rigid; l_0 representing the transfer length.

$$l_0 = \sqrt[4]{\frac{(4E_p I_p)}{E_s}} \tag{7}$$

Where:

E_s is the soil reaction modulus, also known as the frontal pressure mobilization modulus
E_p is the elastic deformation modulus of the pile
I_p is the moment of inertia of the pile

1.1. NUMERICAL MODELING WITH *ROBOT STRUCTURAL ANALYSIS*

Numerical modeling was performed using *Robot Structural Analysis* (*RSA*) structural analysis software in order to compare the analytical results with those obtained from numerical simulation, providing a more in-depth understanding of soil-pile interactions under lateral loads. The Winkler behavior model, used in the analytical approach, is also available on *RSA*. This will allow for a more accurate comparative study, as the analytical and numerical studies are based on the same model. This method is one of the oldest for predicting the lateral reaction modulus of the soil and therefore the displacements and stresses. Using the data presented in Table 1, Figure 10 shows the evolution of the lateral reaction coefficient as a function of pile depth, obtained after discretization by finite elements on *REX-Piles*.
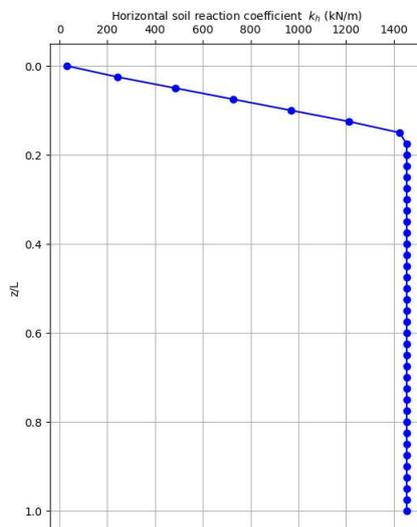

Figure 10. - Evolution of the horizontal soil reaction coefficient (REX-Piles)

As the Winkler elastic soil model is available on *RSA*, after modeling the pile with a diameter of 80 cm and a length of 20 m, we were able to establish a soil model used for bar elements (Figure 11).
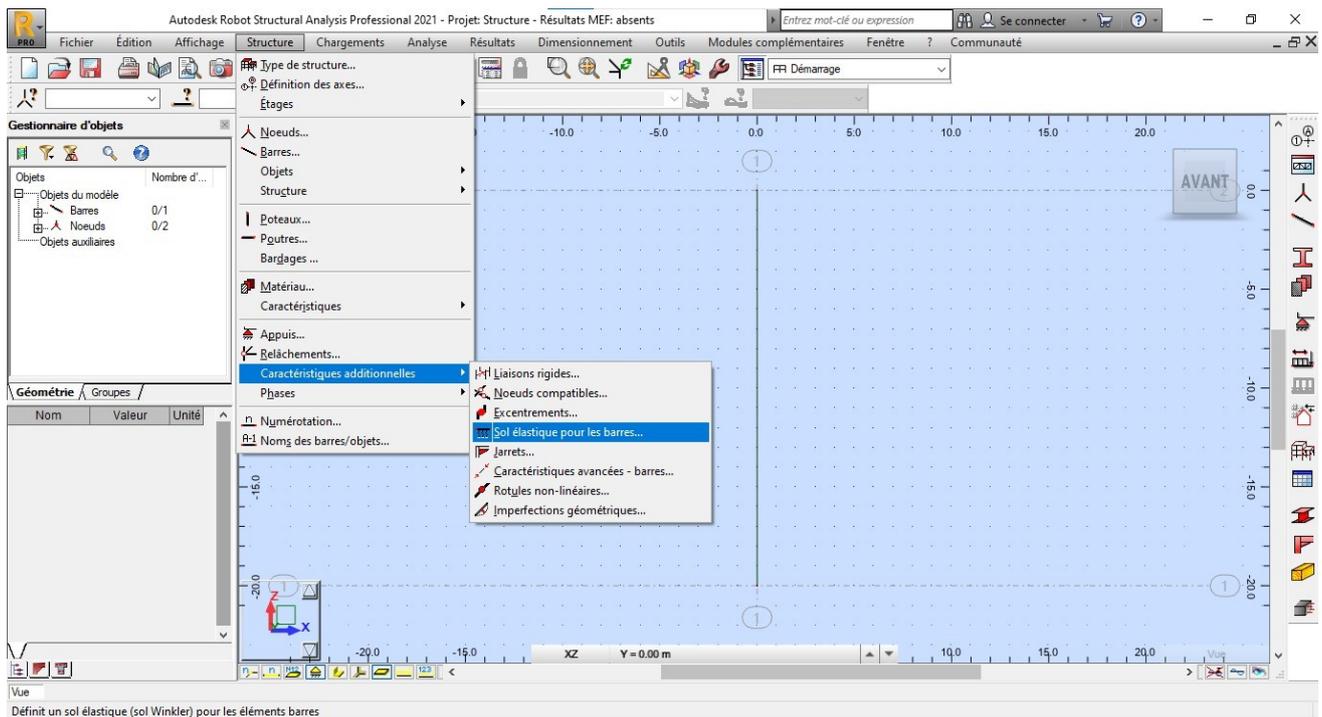

Figure 11. RSA interface for defining Winkler's elastic soil

The soil reaction modulus E_s introduced was determined based on expression *(5)* from **Menard et al.** (Figure 12).
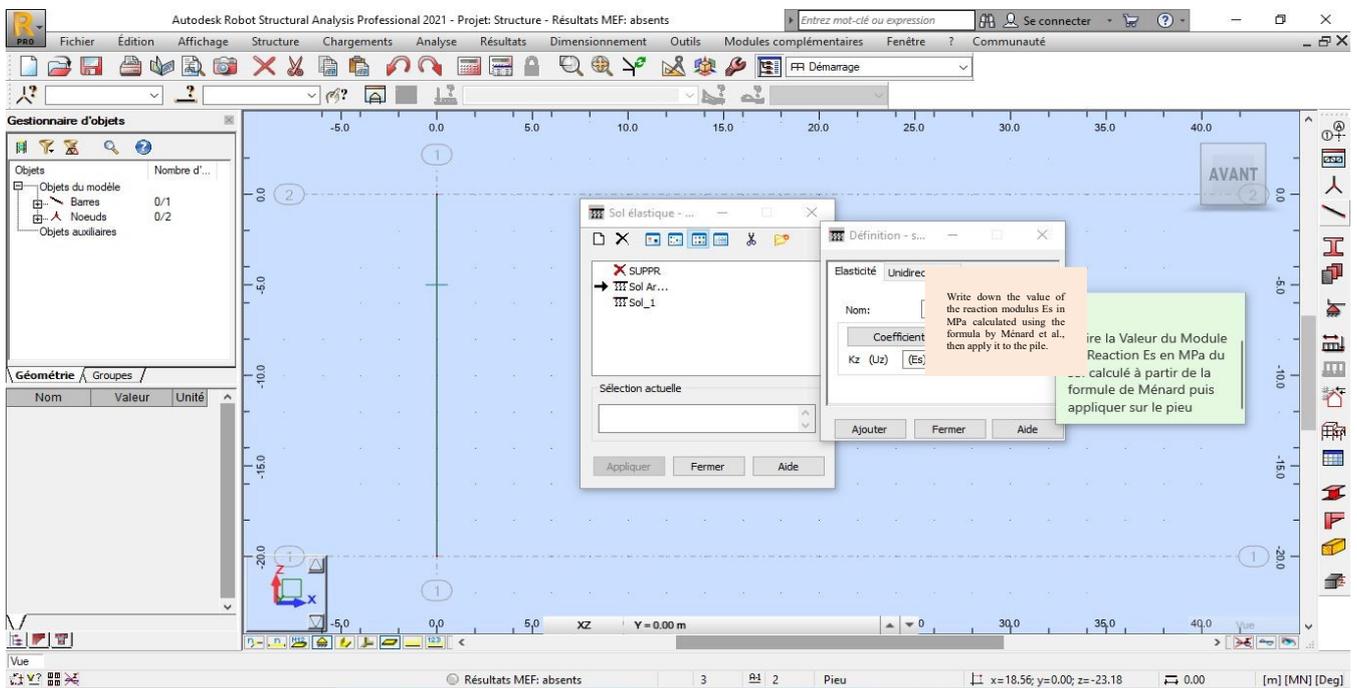

Figure 12. Insertion of the soil reaction modulus value on RSA

The same soil and pile characteristic data and the same applied load values (shear force and head moment) will be used to compare the results between the analytical method and the numerical modeling on *RSA* (Table 2). Figures 13 and 14 show the curves illustrating the displacements, moment, and shear force exerted on the pile compared to the curves obtained from the analytical approach.

Comparison of results obtained : We now seek to compare the results obtained analytically with those generated by the *RSA* software for a flexible pile and a flexible pile subjected at the head to a moment $M_0 = 3$ MN.m and a shear force $V_0 = 0.1$ MN. The lateral displacement y, shear force V, and bending moment M curves are shown in Figures 13 and 14 according to the two models used (analytical and numerical).
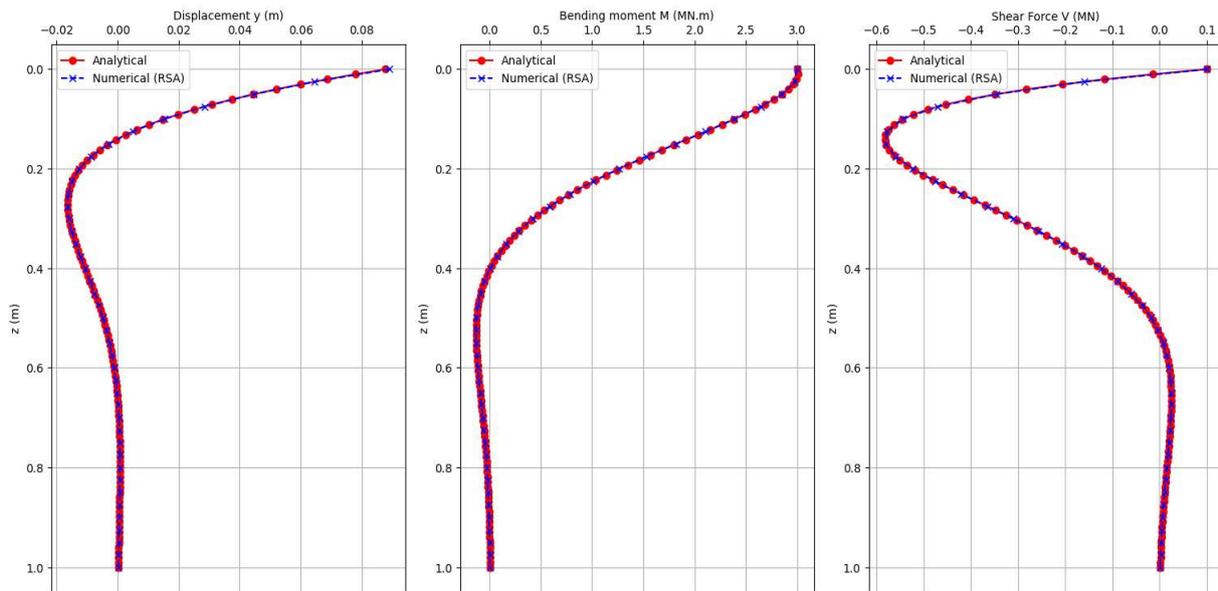
Figure 13. Comparison of the behavior of a flexible pile subjected to lateral loading (Analytical and Numerical)
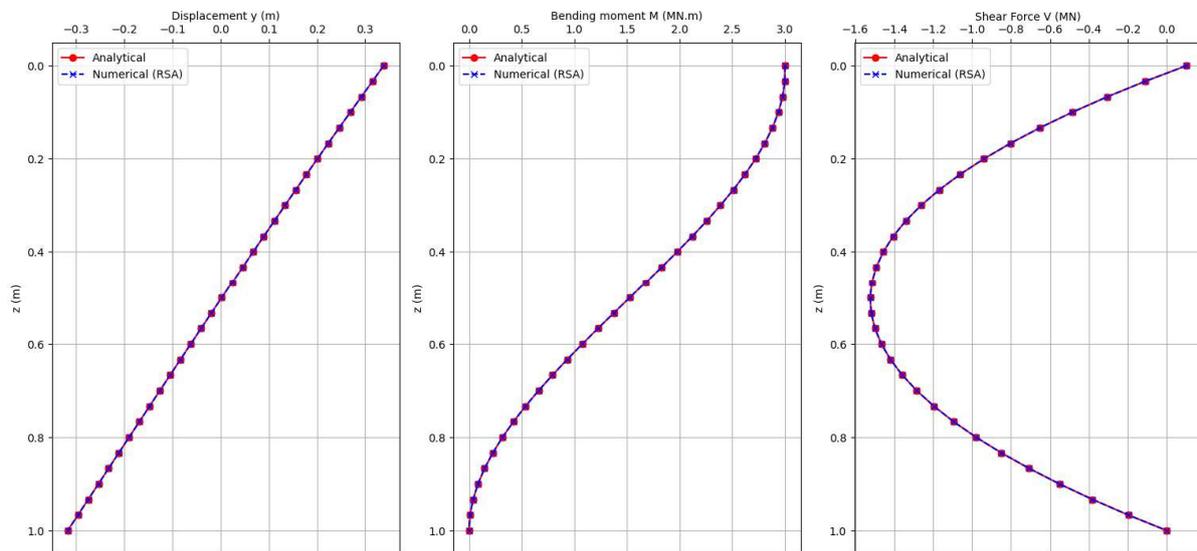

Figure 14. Comparison of the behavior of a rigid pile loaded laterally (Analytical and Numerical)

It can be seen that the patterns of displacement, shear force, and bending moment are almost identical. The results from the Winkler analytical calculation and modeling using *Autodesk Robot Structural Analysis* software for flexible (Figure 13) and rigid (Figure 14) piles show remarkable convergence. This translates into a striking similarity between the values obtained for displacements, maximum moments, and maximum shear forces, highlighting the remarkable accuracy of the results from both approaches. This can be explained by the fact that both calculation methods are based on the behavior model used for a laterally loaded pile, which in this case is Winkler's elastic soil model using the soil reaction modulus E_s value. In the case of flexible piles (Figure 13), however, there is a negligible difference in the maximum displacement value obtained using the two approaches. This shows that the calculation methods, as well as the parameters and input conditions used in both models, are consistent for this type of structure. This consistency reinforces the validity of the models used to analyze the behavior of flexible piles and confirms a certain degree of accuracy in the results obtained. Table 2 shows the percentage convergence of the maximum values obtained using the two study methods, demonstrating the remarkable accuracy of the results.

Table 2. Convergence in % of maximum values according to the two calculation methods (Analytical and Numerical)

Approach	$y_{max}(cm)$		$V_{max}(MN)$		$M_{max}(MN.m)$	
	Rigid	Rigid	Flexible	Rigid	Flexible	Rigid
Analytical	94.22 %	100 %	100 %	100 %	100 %	100 %
Numerical (RSA)						

CONCLUSION

This study presents an analysis of the behavior of piles subjected to axial or lateral loads, focusing on the complex interaction between the soil and the pile. The theoretical and numerical analysis sheds light on the mechanisms by which the forces applied at the head are transferred and balanced by the soil reactions, as well as the mobilization of axial and lateral displacements along the pile.

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with the analytical approach***

Using numerical tools such as Robot Structural Analysis (*RSA*) with its REX-Piles add-on module based on the finite element method, it was possible to perform detailed modeling of soil reactions along the pile. A comparison of the results obtained through analytical resolution and digital modeling reveals similar trends despite a few differences, confirming the reliability and accuracy of both methods and highlighting the importance of choosing the method best suited to the specific design objectives of deep foundations. Finally, this study demonstrates that numerical methods, such as *RSA &REX-Piles*, offer promising solutions for the analysis of soil-foundation interactions and can serve as a basis for optimization in the design of deep foundations.

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