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RESEARCH ARTICLE

APPLICATION OF MOHAND TRANSFORM FOR SOLVING PROBLEMS ON NEWTON'S LAW OF COOLING

*Dr. Lokanath Sahoo

Principal-Cum-Reader in Mathematics, Gopabandhu Science College, Athgarh, Cuttack, Odisha, India

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ABSTRACT

The Newton's Law of Cooling arises in the field of Physics . In this paper , I use Mohand Transform for solving problems on Newton's Law of Cooling and one application is given in order to demonstrate the effectiveness of Mohand Transform for solving problems on Newton's Law of Cooling.

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INTRODUCTION

Newton's Law of Cooling is a differential equation that predicts the cooling of a warm body placed in a cold environment . According to the law, the rate at which the temperature of the body decreases is proportional to the difference of temperature between the body and its environment . In symbols

$$\frac{dT}{dt} = -k\ (T - T_e) \quad (1)$$

with initial condition as $T (t_0) = T_0$ (2)

where T is the temperature of the object,

T_e is the constant temperature of the environment

K is the constant of proportionality,

T_0 is the initial temperature of the object at time t_0.

In equation (1) , the negative sign in the RHS is taken because the temperature of the body is decreasing with time and so the derivative $\frac{dT}{dt}$ must be negative.

MOHAND *TRANSFORM*: The Mohand Transform of the function f(t) is denoted by M{f(t)} and is defined by

Corresponding author:** ***Dr. Lokanath Sahoo,
Principal-cum-Reader in Mathematics, Gopabandhu Science College , Athgarh , Cuttack, Odisha, India.

$M\{f(t)\} = s^2 \int_0^\infty f(t)e^{-st}dt = g(s)$ (3)

Where M is the Mohand Transform operator. The Mohand Transform of the function f(t) for t ≥ 0 exists if f(t) is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mohand Transform without large computational work.

LINEAR PROPERTY OF MOHAND TRANSFORM

M { α f(t) + β g(t) } = α M {f(t)} + β M {g(t)} where α and β are constants.

MOHAND *TRANSFORM OF SOME STANDARD FUNCTIONS*

Sl. No.	f(t)	M{f(t)}=g(s)
1	1	s
2	t	1
3	t^2	$\frac{2!}{s}$
4	t^n, n ∈ N	$\frac{n!}{s^{n-1}}$
5	e^{at}	$\frac{s^2}{(s-a)}$
6	e^{-at}	$\frac{s^2}{(s+a)}$
7	sinat	$\frac{as^2}{(s^2+a^2)}$
8	cosat	$\frac{s^3}{(s^2+a^2)}$
9	sinhat	$\frac{as^2}{(s^2-a^2)}$
10	coshat	$\frac{s^3}{(s^2-a^2)}$

INVERSE MOHAND TRANSFORM

If M { f(t) } = g(s) ,then f(t) is called the inverse Mohand Transform of g(s) and is denoted by $M^{-1}\{g(s)\}$.

MOHAND *TRANSFORM OF DERIVATIVES OF THE FUNCTION f(t)*

If M { f(t) } = g(s) ,then
i) M $\{f'(t)\} = sg(s) - s^2 f(0)$
ii) M {f '' (t)} = $s^2 g(s) - s^3 f(0) - s^2 f'(0)$
iii) M $\{f'''(t)\} = s^3 g(s) - s^4 f(0) - s^3 f'(0) - s^2 f''(0)$
iv) M $\{f^n(t)\} = s^n g(s) - s^{n+1} f(0) - s^n f'(0) - \cdots - s^2 f^{n-1}(0)$

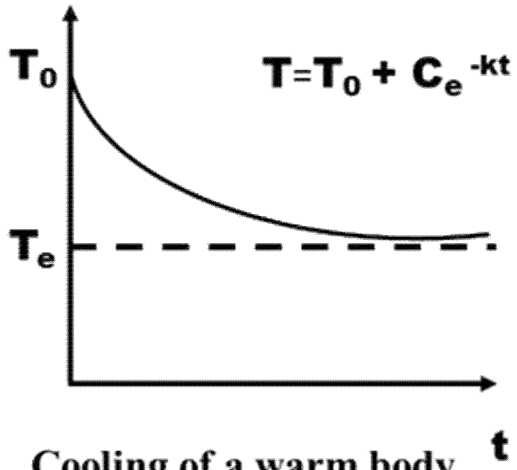
MOHAND *TRANSFORM FOR NEWTON'S LAW OF COOLING*

In this section, I present Mohand Transform for Newton's Law of Cooling given by (1) and (2).Applying Mohand Transform on both sides of (1) , we have

M { $\frac{dT}{dt}$ } = M $\{-k(T-T_e)\}$ (4)

⇨ $sM\{T(t)\} - s^2T(o) = -kM\{T - T_e\}$
⇨ $sM\{T(t)\} - s^2T(o) = -kM\{T(t)\} + kM\{T_e\}$
⇨ $sM\{T(t)\} - s^2T_0 = -kM\{T(t)\} + kT_eM\{1\}$
⇨ $sM\{T(t)\} - s^2T_0 = -kM\{T(t)\} + kT_es$
⇨ $(s+k)M\{T(t)\} = kT_es + s^2T_0$
⇨ $M\{T(t)\} = T_e\frac{ks}{(s+k)} + T_0\frac{s^2}{(s+k)}$
⇨ $M\{T(t)\} = T_e\left\{s - \frac{s^2}{(s+k)}\right\} + T_0\frac{s^2}{(s+k)}$
⇨ $M\{T(t)\} = T_e\{M\{1\} - M\{e^{-kt}\}\} + T_0M\{e^{-kt}\}$
⇨ $M\{T(t)\} = T_eM\{1\} - T_eM\{e^{-kt}\} + T_0M\{e^{-kt}\}$
⇨ $M\{T(t)\} = M\{T_e - T_ee^{-kt} + T_0e^{-kt}\}$
⇨ $T(t) = T_e - T_ee^{-kt} + T_0e^{-kt}$
⇨ $T(t) = T_e + (T_0 - T_e)e^{-kt}$
⇨
⇨ $T(t) = T_e + Ce^{-kt}$ where C = $(T_0 - T_e)$ (5)

Where $T(t)$ *is the temperature of the object at any time t and* T_e *is the temperature of the environment.*



Cooling of a warm body

This function decreases exponentially, but approaches T_e as $t \to \infty$ instead of zero.

APPLICATIONS

In this section, one application is given in order to demonstrate the effectness of Mohand Transfom for solving problems on Newton's Law of Cooling.

***Application*-1**: An apple pie with an initial temperature of 170^o C is removed from the oven and left to cool in a room with an air temperature of 20^o C . Given that the temperature of the pie initially decreases at a rate of 3.0^o C / min. How long will it take for the pie to cool to a temperature of 30^o C ?

Solution: Assuming that the pie obeys Newton's Law of Cooling, we have the following in formations:

$\frac{dT}{dt} = -k\ (T-20)$, T(0) = 170 , $T'(0) = -3.0$

Where T is the temperature of the pie in degree Celsius

t is the time in minutes and k is an unknown constant .

We can find the value of k by putting the information we know about t = 0 directly into the differential equation:

$-3 = -k(170-20)$
=> $k = \frac{1}{50} = 0.02$

So, the differential equation can be written as

$\frac{dT}{dt} = -\frac{1}{50}\ (T-20)$

⇨ $M\left\{\frac{dT}{dt}\right\} = -\frac{1}{50}\ M\{T-20\}$

⇨ $M\left\{\frac{dT}{dt}\right\} = -\frac{1}{50}\ M\{T\} + \frac{2}{5}\ M\{1\}$

⇨ $sM\{T(t)\} - s^2T(0) = -\frac{1}{50}\ M\{T(t)\} + \frac{2}{5}\ s$

⇨ $sM\{T(t)\} - 170s^2 = -\frac{1}{50}\ M\{T(t)\} + \frac{2}{5}\ s$

⇨ $\left(s + \frac{1}{50}\right) M\{T(t)\} = \frac{2}{5}s + 170\ s^2$

⇨ $M\{T(t)\} = \frac{\frac{2}{5}s}{\left(s+\frac{1}{50}\right)} + \frac{170s^2}{\left(s+\frac{1}{50}\right)}$

⇨ $M\{T(t)\} = \frac{20}{50}\frac{s}{\left(s+\frac{1}{50}\right)} + 20\frac{s^2}{\left(s+\frac{1}{50}\right)} + 150\frac{s^2}{\left(s+\frac{1}{50}\right)}$

⇨ $M\{T(t)\} = 20\ \frac{s}{\left(s+\frac{1}{50}\right)}\left(\frac{1}{50} + s\right) + 150\frac{s^2}{\left(s+\frac{1}{50}\right)}$

⇨ $M\{T(t)\} = 20s + 150\frac{s^2}{\left(s+\frac{1}{50}\right)}$

⇨ $M\{T(t)\} = 20M\{1\} + 150M\{e^{-\frac{1}{50}t}\}$

⇨ $M\{T(t)\} = M\left\{20 + 150e^{-\frac{1}{50}t}\right\}$

⇨ $T(t) = 20 + 150e^{-\frac{1}{50}t}$ (6)

Putting T=30 in (6) ,we get

$30 = 20 + 150e^{-\frac{1}{50}t}$

⇨ $e^{-\frac{1}{50}t} = \frac{1}{15}$

⇨ $e^{\frac{1}{50}t} = 15$

⇨ $\frac{1}{50}\,t = \ln 15$

⇨ $t = 50\,ln15 = 50 * 2.7080502011 = 135.4\,min$

Hence it will take 135.4 minutes for the pie to cool to a temperature of 30^{o} C .

CONCLUSION

In this paper , I have successfully developed the Mohand Transform for solving problems on Newton's Law of Cooling.The given applications show the effectness of Mohand Transform for solving problems on Newton's Law of Cooling.

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