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RESEARCH ARTICLE

UNSTEADY MHD FREE CONVECTION WITH RADIATION AND JOULE'S HEATING EFFECT ON VERTICAL STRETCHING PLATE ENTRENCHED IN A POROUS MEDIUM

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ARTICLE INFO	ABSTRACT
<i>Article History:</i> Received 17 th October, 2020 Received in revised form 09 th November, 2020 Accepted 21 st December, 2020 Published online 30 th January, 2021	MHD flows given to be unsteady with free convection, in addition toheat transfer through stretching plate held vertically in medium with permeable nature, facing radiation and Joule's effect have been examined. Under usual boussinesq approximation, governing equations pertaining to motion and energy have been determined and converted to first order ordinary differential equations applying similarity transformation. Solvedthese equations numerically usingNewton's iterative method as well as Runge-Kutta method. Computed the values of velocity including temperature taking help of Matlab. Analyzed through graphs the impact of Reynolds, Grashof, Hartmann and Eckert numbers, and Radiation and permeability parameters on the velocity as well as temperature field.
Key words:	
Unsteady, free convection, MHD, stretching vertical plate, Radiation effect, Joule's heating.	

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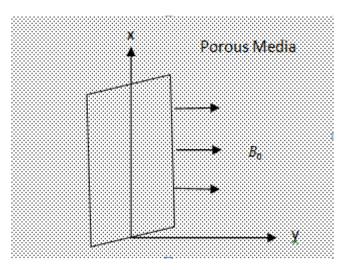
INTRODUCTION

In present era the examination of free convection MHD flows in permeable medium is of incredible thought and has been of much concern for numerous scientists, for their applications in the science and innovation. At high temperature and under the application of magnetic field, flowof electrically conducting fluid has beenstudied and explained in so far as their application in the creation of energy, sun oriented board innovation, atomic building, space vehicle re-emergence, and different mechanical territories is concerned. The outcome and results of the above study haveproved to be of great value and appreciable use in various enterprises. Sacheti et al. [16], got an explicit solution to the free convection flow of MHD given to be unsteady in nature, upon a vertical plate which started impulsively, with heat flux taken to be constant. Crane [10] found a distinct solution of the problem by analyzing flow of a permeable, incompressible fluid, concerning boundary layer, remaining consistent, inside a stretching, having elastic flat sheet, inside possessed plane, changing directly along the length from a confined point.

Pop and Na [17] analyzed a flow, said to be unsteady, starting impulsively from static position, past a wall and it was found that with the lapse of long period of time unsteady flow turns to a steady flow. Elbashbeshy and Bazid[6] explained that unsteadiness parameter affected thickness and momentum of boundary layer. Laha et al. [11] contemplated the property exhibited by three dimensional flow of permeable fluid given to be incompressible, to affect transfer of heat, on a stretching sheet. Afzal [15] researched on transfer of heat utilizing stretching surface. Ishak [9] has explained the characteristic of heat transfer of MHD flow, over a stretching plate given to be unsteady. Govardhan and Kishan [22] examined MHD flow stated to be unsteady, of boundary layer, pertaining to incompressible, micropolar fluid on a stretching sheet. A parameter named Rosseland approximation was clarified by Sparrow [5], by which radiation heat flux has been described in the energy equation. Takhar *et al.* [7] researched free convection flow of MHD, past a vertical plate, semi-infinite during radiating gas. Das *et al.* [21] analyzed the impact of radiation on a flow past an impulsively started vertical plate infinite and flat. Raptis [1] continues to contemplate free convection flow and radiation using a permeable medium. Raptis and Perdikis [19] have gone into the impact of radiation through extraordinary permeable medium on unsteady flow. Characteristics of a vertical plate inserted from a permeable medium with free convection as well as radiation were considered by Badruddin *et al.* [8].

Free convection flow taken to be unsteady, of MHD, over vertical plate which is infinite is continued to be examined in the incorporation of radiation by Perdikis and Raptis [4]. El-Aziz [12] has researched into flow over stretching sheet along with heat transfer over a stretching sheet given to be unsteady, in the context of radiation. Sharma and Choudhary [18] explored the issue of free convective flow of MHD given to be unsteady, past a vertical permeable plate encased in a permeable medium with heat source and existing radiation. Viscous dissipation overa stretching surface happened to be unsteady, under externally applied magnetic field, and has been examined by Abel et al. [14]. Free convective flow of MHD, stated to be unsteady, past a vertical plate being infinite, under heated state, in corporation with viscous dissipation and existing radiation, in a permeable medium, having been provided with time dependent suction was examined by Isreal-Cookev et al. [3]. Anjai Devi and B. Ganga [20] have worked on the issue ofviscous flow pertaining to MHD as well as joule's dissipation in addition to transfer of heat and mass past a permeable stretching surface, part of medium which is permeable. Dessie and Kishan [23] initiated the probe into boundary layer flow concerning MHD as well as transfer of heat of the fluid having changing viscosity, over a stretching sheet within a permeable medium, considering the impact ofviscous dissipationas well as heat source/sink. Analytical solutions for convective flow of MHD given to be of unsteady nature, and transfer of heat and mass along a vertical plate stated to be permeable, under the impact of chemical reaction as well as thermal radiation were reported by Balla and Kishan [24]. The motivation behind the present investigation is to inspect the impact of radiative heating, effect of Joule's heating and stretching on the plate with free convective flow pertaining to MHD as well as transfer of heat in a stretching vertical impermeable plate install in permeable medium.

Formulation of the Problem: The problem is modeled to dissect MHD flowgiven to be unsteady and with free convection, in addition to effect of heat transfer through a stretching impermeable vertical plateinserted in permeable medium, facingjoule's effect with radiation (Figure-1). A vertical plate of the semi-infinite nature is held in a permeable medium which is saturated with electrically conducting fluid which is viscous having the property of being incompressible. X-axis is taken in the direction of the stretching plate and y-axis is represented perpendicular to it. Normal to the surface of the plate, a regular magnetic field $(0, B_0, 0)$ is working. The vertical plate installed in permeable medium is exposed to an unsteadily stretching along the length and a time dependent temperature persists on the plate.





Continuity equation for viscous incompressible fluid is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

According to Boussinesq's approximation, equation of motion is given as

Energy equation is given by

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho} u^2 \uparrow \qquad (3)$$

The boundary conditions are

$$y = 0: u = cx, v = 0, T = T_w$$

$$y \to \infty: u = 0, T = T_{\infty}$$
(4)

u and v are velocity components in the x and y-directions, v the kinematic viscosity, g the acceleration due to gravity, β the volumetric expansion coefficient, κ the thermal conductivity, *K* the permeability of porous medium, q_r the radiative heat flux, and *T* the fluid temperature.

As per Rosseland approximation for radiative heat flux [Brewster, (13)]

$$q_{\rm r} = -\frac{4\sigma}{3\delta} \frac{\partial T^4}{\partial y} \tag{5}$$

 σ is the Stefan – Boltzmann constant and δ is the mean absorption coefficient. It is assumed that the temperature contrasts inside the flow are adequately little with the end goal that T^4 might be communicated as a linear function of the temperature.

By Taylor series expansion of T^4 , neglecting terms of T with higher powers, we have:

$$T^4 \approx 4TT_{\infty}^3 - 3T_{\infty}^4$$
(6)

METHOD OF SOLUTION

Solving (2) and (3) introducing following similarity transformations

Dimensionless forms of equations of motion and energy are

$$(1-\xi)(-\frac{1}{2}\eta f'' + \xi \frac{\partial f'}{\partial \xi}) + \xi(f')^2 - \xi f f'' = (\frac{Gr}{R_e^2})\xi \theta + f''' - (S+M)\xi f'$$
(8)

$$(1-\xi)\left(-\frac{\eta}{2}\theta'+\xi\frac{\partial\theta}{\partial\xi}\right)+\xi(nf'\theta-f\theta')=(P_r^{-1}+\alpha)\theta''+ME_c\xi(f')^2$$
(9)

where, R_e the Reynolds number, Gr the Grashof number, S the permeability parameter, M the magnetic parameter, P_r the Prandtl number, E_c the Eckert number and α the Radiation parameter are defined by

Corresponding boundary conditions:

$$\eta = 0: f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1$$
(11a)

$$\eta \to \infty : f'(\eta) = 0, \ \theta(\eta) = 0 \tag{11b}$$

IV. NUMERICAL METHOD

Equation (8) represents motion and (9) energy. Both are partial differential equations with variable ξ and η .

Case-1

When $\xi = 0$ i.e. at t=0 the equations (8) and (9) reduces to

$$f''' + \frac{\eta}{2} f'' = 0$$
(12)

$$\theta'' + \frac{1}{(P_r^{-1} + \alpha)} \frac{\eta}{2} \theta' = 0$$
(13)

Solution of the equations (12) and (13) are given by

$$f'(\eta) = -erf(\eta) + 1$$

$$\theta(\eta) = -erf(\eta\sqrt{a}) + 1$$
where $a = \frac{1}{4(P_r^{-1} + \alpha)}$
Case-2.

When $\xi = 1$ i.e. $t \to \infty$, the equations (8) and (9) reduce to

In order to solve equations (16) and (17), these are changed into first order ordinary differential equation by taking $f(\eta) = F_0$, $f'(\eta) = F_1$, $f''(\eta) = F_2$, $\theta(\eta) = \Theta_0$, $\theta'(\eta) = \Theta_1$. These are changed as follows:

$$\frac{dF_0}{d\eta} = F_1 \tag{18}$$

$$\frac{dF_1}{d\eta} = F_2 \tag{19}$$

$$\frac{d\Theta_0}{d\eta} = \Theta_1 \tag{21}$$

$$\frac{d\Theta_1}{d\eta} = \frac{1}{P_r^{-1} + \alpha} [bF_1\Theta_0 - F_0\Theta_1 - ME_c(F_1)^2]$$
(22)

subject to the boundary conditions

$$F_0(0) = 0 \ F_1(0) = 1 \ \Theta_0(0) = 1$$

$$F_1(\infty) = 0 \ \Theta_0(\infty) = 0$$
.....(23)

Now, the equations (18) to (22) form first order ordinary differential equations whose solution is to be obtained applying Runge-Kutta method. For the proper implementation of which five initial conditions given here under have to be satisfied.

 $F_0(0)$, $F_1(0)$, $F_2(0)$, $\Theta_0(0)$ and $\Theta_1(0)$ are required. But $F_2(0)$ and $\Theta_1(0)$ are not known, therefore they must be first obtained and refined to approximate the boundary condition.

The computational technique stated by Goldstein [2] is applied. For different values of parameters profiles of velocity and temperature are obtained numerically using Matlab program and shown through figures.

Skin Friction

Mathematically shearing stress at the plate is represented by,

$$C_f = \frac{\iota}{(\rho U^2 / 2)} \tag{24}$$

Table 1. Values of - C

It is proportional to $f''(\eta)$

For non-dimensional parameters values of skin friction coefficient (C_{f})

are presented in Table 1.

Gr	Re	S	М	Ec	С	C_{f}
10	10	1	2	0.5	2	-1.9721
20	10	1	2	0.5	2	-1.9420
30	10	1	2	0.5	2	-1.9120
10	20	1	2	0.5	2	-1.9947
10	30	1	2	0.5	2	-1.9989
10	10	0.5	2	0.5	2	-1.8432
10	10	1.5	2	0.5	2	-2.0934
10	10	1	4	0.5	2	-2.4210
10	10	1	6	0.5	2	-2.8010
10	10	1	2	0.2	2	-1.9736
10	10	1	2	0.8	2	-1.9705
10	10	1	2	0.5	4	-1.9756
10	10	1	2	0.5	6	-1.9779

Nusselt Number

$$Nu = \frac{hL}{\kappa}$$

and is proportional to $-\theta'(\eta)$. Its values for (non-dimensional) physical parameters are exhibited in Table 2.

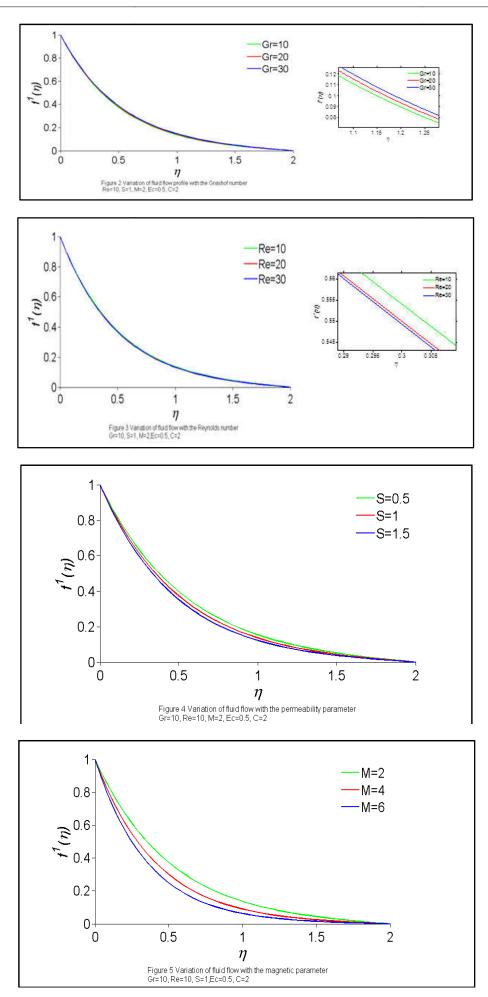
Gr	Re	S	Μ	Ec	С	Nu
10	10	1	2	0.5	2	1.0775
20	10	1	2	0.5	2	1.0783
30	10	1	2	0.5	2	1.0789
10	20	1	2	0.5	2	1.0769
10	30	1	2	0.5	2	1.0768
10	10	0.5	2	0.5	2	1.0796
10	10	1.5	2	0.5	2	1.0744
10	10	1	4	0.5	2	0.6063
10	10	1	6	0.5	2	0.2269
10	10	1	2	0.2	2	1.3934
10	10	1	2	0.8	2	0.7611
10	10	1	2	0.5	4	1.3874
10	10	1	2	0.5	6	1.6289

Table 2. Values of- Nu

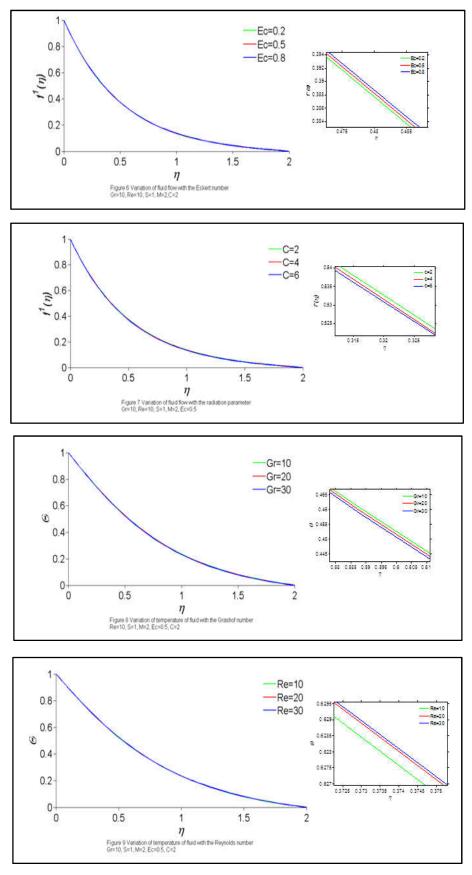
RESULTS AND DISCUSSION

Due to numerical computations, dimensionless velocity and temperature circulations towards the flow under consideration are gotten for varieties while in the administering parameters viz., the thermal Grashof number Gr, Reynolds number Re, permeability parameter S, magnetic field parameter M, Eckert number Ec, radiation parameter C. In the present study, the foregoing default parametric values are adopted. Gr = 10.0, Re = 10.0, S=1, M = 2.0, Pr = 0.71, Ec=0.5, C = 2.0. In Fig. 2. Impact on velocity by thermal buoyancy force parameter Gr can be seen and it is observed that velocity is increases due to this force. It may be seen with the figure, the effect of thermal buoyancy increases velocity of fluid. It is clear from fig. 4 that when permeability parameter S is increased velocity decreases that proves it to be inversely proportional to K (permeability coefficient).

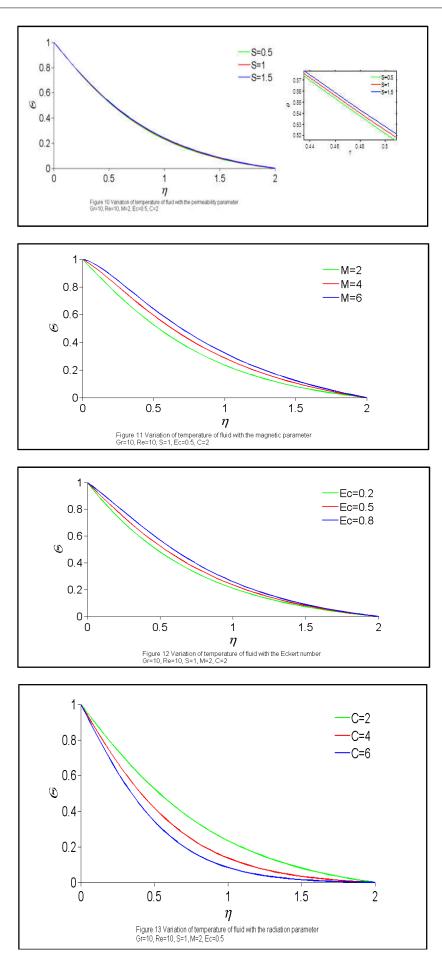
.....(25)



A permeable medium with low permeability coefficient will restrict the movement of fluid through it. Fig. 5 contains velocity evaluation for different values of M the Magnetic parameter. it is noticed that velocity decreases when *M* is increased. This can be the result of drag, produced by magnetic field acting on the electrically conducting fluid and is known as Lorentz force which acts against the velocity of fluid and retards it. From Fig. 3, Fig. 6 and Fig. 7 it is observed that the impact of Reynolds number, Eckert number and Radiation parameter are less productive on velocity profile.



From Fig. 11 we notice that the impact of Lorentz force since it act inverse to flow direction produces a sort of friction in the flow, this friction successively created more heat energy which in the long run improves the distribution of temperature of the flow.



The influence of Eckert number in the temperature fluid is exhibited in Fig. 12. Ascend in Eckert number causes enrichment inside the kinetic energy, thus the improvement in average kinetic energy consequences of how the temperature on the fluid upgrades. It very well may be seen that temperature having a place with the fluid is expanded when the Eckert number increases. Fig.13 infers

that when the radiation parameter increments the temperature faces abetments. Fig. 8, Fig. 9 and Fig. 10 guarantees that for little varieties from the valuation of Grash of number, Reynolds number and permeability parameter are less viable in the temperature profiles. Table 1. From this table it is noticed that when Grash of and Eckert numbers increase there is increase in skin friction. Skin friction reduces while using the increasing value of Reynolds number, permeability parameter, magnetic parameter and Radiation parameter. From Table 2, it happens to be seen that this Nusselt number increments while utilizing the expansion in estimation of Grashof number along with Radiation parameter. Nusselt number abatements for a rising value of Reynolds number, permeability parameter, magnetic parameter and Eckert number.

Conclusion

- Increased magnetic parameter,, the flow velocity is hindered and fluid temperature increases that are in great understanding while at the same time using the physical phenomenon of Lorezian force.
- The illustrative nature of this fluid flow impeded when permeability parameter of this medium is increased.
- The fluid temperature increases while Eckert number gets increaseand increment in radiation parameter brings about reduction in fluid temperature.
- The skin friction reduces while utilizing the increasing valuation on Reynolds number, permeability parameter, Radiation and magnetic parameters while increase in the valuation on Grashof and Eckert numbers gives increase to skin friction.
- There is increase in Nusselt number by increasingGrashof number and Radiation parameter while Nusselt number reduces while using the increasing valuation on Reynolds number, permeability parameter, magnetic parameter and Eckert number.

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