Available Online at http://www.journalajst.com

ISSN: 0976-3376

ASIAN JOURNAL OF
SCIENCE AND TECHNOLOGY

Asian Journal of Science and Technology
Vol. 11, Issue, 05, pp.10959-10968, May, 2020

RESEARCH ARTICLE

STATE SPACE APPROACH TO BOUNDARY VALUE PROBLEM FOR DOUBLE POROUS VISCOELASTIC MEDIUM

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ARTICLE INFO

Article History:
Received 07th February, 2020
Received in revised form
19th March, 2020
Accepted 14th April, 2020
Published online 30th May, 2020

Key words:

Double porosity, viscoelasticity, State space approach, Thermomechanical sources.

ABSTRACT

The present paper deals with a boundary value problem in a homogeneous, isotropic double porous viscoelastic medium subjected to thermomechanical sources. After developing mathematical formulation, a state space approach has been applied to investigate the problem. As an application of the approach, normal force and thermal source have been taken to illustrate the utility of the approach. The expressions for the components of normal stress, equilibrated stress and the temperature change are obtained in the frequency domain and computed numerically. Numerical simulation is prepared for these quantities and simulated results for these quantities are depicted graphically for a particular model. Some particular cases of interest are also deduced from the present investigation.

Citation: Rajneesh Kumar, Aseem Miglani, Sonika Chopra and Monika Kalra. 2020. "State space approach to boundary value problem for double porous Viscoelastic Medium", *Asian Journal of Science and Technology*, 11, (05), 10959-10968.

INTRODUCTION

The linear theory of viscoelasticity described by the linear behaviour of both elastic and non-elastic materials, provide a basis for describing the attenuation of seismic waves. Before 1960 most of the work on linear viscoelastic wave propagation for which explicit solutions were obtained was essentially for dimensional and for specific material. Bland (1) has given account of three dimensional linear viscoelasticity theories. He concluded that as in perfectly elastic isotropic medium, under assumption of small displacement, two types of waves can be propagated in an isotropic viscoelastic medium when body forces are absent. The Kelvin-Voigt model is one of the macroscopic mechanical models often used to describe the viscoelatic behaviour of a material. Iesan and Scalia (2) proved some theorems in the theory of thermoviscoelasticity. Othman et al. (3) studied the generalized thermoviscoelastic plane waves propagation with two relaxation times. Many branches of engineering including material science, petroleum industry, chemical engineering, biomechanics and other such fields of engineering include theories of porous media. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the concrete body is deformed. However the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behaviour of a fluid saturated porous thermoelastic medium becomes very complicated. So researchers from time to time, have tried to overcome this difficulty and we see many porous medium theories in the literature. A brief historical background of these theories is given by de Boer (4, 5). Biot (6) proposed a general theory of three-dimensional deformation of fluid saturated porous solids. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen (7), de Beor and Ehlers (8) in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modelled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the requirement of many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. (9), where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers. The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super-Saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (Sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). Wilson and Aifantis (10) presented the theory of consolidation with the double porosity. Khaled, Beskos and Aifantis (11) employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidations with double porosity. Wilson and Aifantis (12) discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yield three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space. Beskos and Aifantis (13) presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan (14) studied the unified theory of flow and deformation in double porous media.

Kumar and Kumar (15) investigated the wave propagation at the boundary surface of elastic and initially stressed viscothermoelastic diffusion with voids. Sharma and Kumar (16) studied the propagation of plane waves and fundamental solution in thermoviscoelatsic medium with voids. Kumar *et al* (17) investigated the fundamental solution in micropolar viscothermoelastic solids with voids. Svanadze(18) Studied the problem of plane waves and steady vibrations in the theory of viscoelasticity for Kelvin-Voigt material with double porosity. Svanadze (19-21) investigated some problems on elastic solids and thermoelastic solids with double porosity. In recent years the state space description of linear system has been used extensively in various areas of engineering, such as the analysis of control systems. The state space approach offers an attractive way to avoid the difficulties of the traditional linear model approach. Bahar and Hetnarski investigated good number of problems in thermoelasticity by using state space approach (22-27). Sharma (28) studied the one dimensional problems in generalized theories of thermoelasticity subjected to heat source and body forces by using state space approach. Othman *et al* (29) established the model of the two-dimensional generalized thermo-viscoelasticity with two relaxation times and used normal mode analysis to obtain the exact expressions for the temperature distribution, thermal stresses and the displacement components. Ezzat *et al* (30) applied state space approach to generalized thermo-viscoelasticity with two relaxation times. Maghraby *et al* (31) used the state space approach to the one dimensional problem of thermoelasticity with two relaxation times. Youssef and Al-Lehaibi (32) considered a half-space filled with an elastic material and used state space approach to obtain the general solution for any set of boundary conditions. Youssef and Harby (33) considered an infinite elastic body with a spherical cavity and constant elastic parameters, and used state space technique to obtain the general solution for any set of boundary conditions. Sherief and El-Sayed (34) applied the state space approach to two-dimensional generalized micropolar thermoelasticity. Kumar *et al* (35) studied a boundary value problem for thermoelastic material with double porosity using state space approach. Kumar and Vohra (36) investigated the plane deformation in elastic materials with double porosity using state space approach.

In the present paper, the state space approach has been used to solve a boundary value problem for a double porous viscoelastic medium. The expressions for normal stress, equilibrated stress and temperature distribution are obtained in closed form, computed numerically and represented graphically for normal force and thermal source to show the effect of viscosity. Some particular cases of interest are also deduced from the present investigation.

Basis Equations

Following Iesan and Quintanilla (37), the constitutive relations and field equations for homogeneous double porous viscoelastic medium without body forces, extrinsic equilibrated body forces and heat sources can be written as:

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi - \beta\delta_{ij}T, \tag{1}$$

$$\sigma_i = \alpha\varphi,_i + b_1\psi,_i , \tag{2}$$

$$\tau_i = b_1\varphi,_i + \gamma\psi,_i , \tag{3}$$

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla.\vec{u} + b\nabla\varphi + d\nabla\psi - \beta\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{4}$$

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - b\nabla.\vec{u} - \alpha_1\varphi - \alpha_3\psi + \gamma_1 T = \kappa_1\frac{\partial^2\varphi}{\partial t^2}, \tag{5}$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - d\nabla.\vec{u} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T = \kappa_2\frac{\partial^2\psi}{\partial t^2}, \tag{6}$$

$$\beta T\,\frac{\partial e}{\partial t} + \gamma_1 T_0\frac{\partial\varphi}{\partial t} + \gamma_2 T_0\frac{\partial\psi}{\partial t} + \rho C^*\frac{\partial T}{\partial t} = K^*\frac{\partial^2 T}{\partial t^2}\ , \tag{7}$$

where λ and μ are Lame's constants, ρ is the mass density; $\beta = (3\lambda + 2\mu)\alpha$; α is the linear thermal expansion; C^* is the specific heat at constant strain, u_i are the displacement components; t_{ij} is the stress tensor; k_1 and k_2 are coefficients of equilibrated inertia; v_1 is the volume fraction field corresponding to pores and v_2 is the volume fraction field corresponding to fissures; φ and ψ are the volume fraction fields corresponding to v_1 and v_2 respectively; σ_1 is the equilibrated stress corresponding to v_1; τ_1 is the equilibrated stress corresponding to v_2; k is the coefficient of thermal conductivity and $b,\ d,\ b_1,\ \gamma,\ \gamma_1,\ \gamma_2$ are constitutive coefficients; δ_{ij} is the Kronecker delta; Δ is the Laplacian operator and T is the temperature change measured from the absolute temperature $T_0 (T_0 \neq o)$; a superposed dot represents differentiation with respect to time variable t. Assuming the viscoelastic nature of the material described by Voigt (38) model of linear viscoelasticity, we replace the porous thermoelastic constants $\beta,\ \lambda,\ \mu,\ b,\ d,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \gamma_1,\ \gamma_2,\ \alpha,\ \gamma,\ b_1$ by

β_ε, λ_ε, μ_ε, b_ε, d_ε, $\alpha_{1\varepsilon}$, $\alpha_{2\varepsilon}$, $\alpha_{3\varepsilon}$, $\gamma_{1\varepsilon}$, $\gamma_{2\varepsilon}$, α_ε, γ_ε, $b_{1\varepsilon}$ respectively in equations (1)-(7) and, we obtain

$$t_{ij} = \lambda_\varepsilon e_{rr}\delta_{ij} + 2\mu_\varepsilon e_{ij} + b_\varepsilon \delta_{ij}\varphi + d_\varepsilon \delta_{ij}\psi - \beta_\varepsilon \delta_{ij} T, \quad (8)$$

$$\sigma_i = \alpha_\varepsilon \varphi,_i + b_{1\varepsilon}\psi,_i, \quad (9)$$

$$\tau_i = b_{1\varepsilon}\varphi,_i + \gamma_\varepsilon \psi,_i, \quad (10)$$

$$\mu_\varepsilon \nabla^2 \vec{u} + (\lambda_\varepsilon + \mu_\varepsilon)\nabla\nabla.\vec{u} + b_\varepsilon \nabla\varphi + d_\varepsilon \nabla\psi - \beta_\varepsilon \nabla T + F_i = \rho \frac{\partial^2 \vec{u}}{\partial t^2}, \quad (11)$$

$$\alpha_\varepsilon \nabla^2\varphi + b_{1\varepsilon}\nabla^2\psi - b_\varepsilon \nabla.\vec{u} - \alpha_{1\varepsilon}\varphi - \alpha_{3\varepsilon}\psi + \gamma_{1\varepsilon}T = \kappa_1 \frac{\partial^2\varphi}{\partial t^2}, \quad (12)$$

$$b_{1\varepsilon}\nabla^2\varphi + \gamma_\varepsilon \nabla^2\psi - d_\varepsilon \nabla.\vec{u} - \alpha_{3\varepsilon}\varphi - \alpha_{2\varepsilon}\psi + \gamma_{2\varepsilon}T = \kappa_2 \frac{\partial^2\psi}{\partial t^2}, \quad (13)$$

$$\beta_\varepsilon T_0 \frac{\partial e}{\partial t} + \gamma_{1\varepsilon}T_0 \frac{\partial\varphi}{\partial t} + \gamma_{2\varepsilon}T_0 \frac{\partial\psi}{\partial t} + \rho C^* \frac{\partial T}{\partial t} = K^* \frac{\partial^2 T}{\partial t^2}, \quad (14)$$

where

$$\lambda_\varepsilon = \lambda + \lambda_v \frac{\partial}{\partial t}, \qquad \mu_\varepsilon = \mu + \mu_v \frac{\partial}{\partial t},$$

$$b_\varepsilon = b + b_v \frac{\partial}{\partial t}, \qquad d_\varepsilon = d + d_v \frac{\partial}{\partial t},$$

$$\alpha_{1\varepsilon} = \alpha_1 + \alpha_{1v}\frac{\partial}{\partial t}, \qquad \alpha_{2\varepsilon} = \alpha_2 + \alpha_{2v}\frac{\partial}{\partial t},$$

$$\alpha_{3\varepsilon} = \alpha_3 + \alpha_{3v}\frac{\partial}{\partial t}, \qquad \alpha_\varepsilon = \alpha + \alpha_v \frac{\partial}{\partial t},$$

$$\gamma_\varepsilon = \gamma + \gamma_v \frac{\partial}{\partial t}, \qquad b_{1\varepsilon} = b_1 + b_{1v}\frac{\partial}{\partial t},$$

in which λ_ε, μ_ε, b_ε, d_ε, $\alpha_{1\varepsilon}$, $\alpha_{2\varepsilon}$, $\alpha_{3\varepsilon}$, α_ε, γ_ε, $b_{1\varepsilon}$ are the viscoelastic constants.

Formulation and solution of the problem: A homogeneous, isotropic thermoelastic solid with double porosity structure occupying the region $0 \le x < \infty$, whose state variable depend only on the space variables distance x and time t, has been considered for which the displacement components $u_i (i = 1,2,3)$, volume fraction φ and ψ, and temperature change T are taken as

$$u_i = u(x,t), \qquad \varphi = \varphi(x,t),$$
$$\psi = \psi(x,t), \qquad T = T(x,t)$$

(15)

Now, we define the dimensionless quantities as

$$x' = \frac{\omega}{c_1}x, \quad u' = \frac{\omega}{c_1}u, \quad \varphi' = \frac{k_1\omega^2}{\alpha_1}\varphi,$$

$$\psi' = \frac{k_1\omega^2}{\alpha_1}\psi, \quad T' = \frac{T}{T_0}, \quad \sigma_1' = \frac{c_1}{\alpha\omega}\sigma_1,$$

$$\tau_1' = \frac{c_1}{\alpha\omega}\tau_1, \quad t' = \omega t, \quad t_{ij}' = \frac{t_{ij}}{\beta T_0}, \quad (16)$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \qquad \omega = \frac{\rho C^* c_1^2}{K^*},$$

Here ω and c_1 are the constants having dimensions of frequency and velocity in the medium respectively.

Making use of the dimensionless quantities given in (16) in equations (11)-(14), we get

$$\frac{\partial^2 u}{\partial x^2} + \delta_1 \frac{\partial\varphi}{\partial x} + \delta_2 \frac{\partial\psi}{\partial x} - \delta_3 \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \quad (17)$$

$$\delta_4 \frac{\partial^2\varphi}{\partial x^2} + \delta_5 \frac{\partial^2\psi}{\partial x^2} - \delta_6 \frac{\partial u}{\partial x} - \delta_7\varphi - \delta_8\psi + \delta_9 T = \frac{\partial^2\varphi}{\partial t^2}, \quad (18)$$

$$\delta_{10} \frac{\partial^2\varphi}{\partial x^2} + \delta_{11} \frac{\partial^2\psi}{\partial x^2} - \delta_{12} \frac{\partial u}{\partial x} - \delta_{13}\varphi - \delta_{14}\psi + \delta_{15} T = \frac{\partial^2\psi}{\partial t^2}, \quad (19)$$

$$\delta_{16} \frac{\partial^2 T}{\partial x^2} - \delta_{17} \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) - \delta_{18} \frac{\partial\varphi}{\partial t} - \delta_{19} \frac{\partial\psi}{\partial t} = \frac{\partial T}{\partial t}, \quad (20)$$

where

$$\delta_1 = \frac{b\alpha_1}{\rho c_1^2 k_1 \omega^2}, \quad \delta_2 = \frac{d\alpha_1}{\rho c_1^2 k_1 \omega^2}, \quad \delta_3 = \frac{\beta T_0}{\rho c_1^2}, \quad \delta_4 = \frac{\alpha}{c_1^2 k_1},$$

$$\delta_5 = \frac{b_1}{c_1^2 k_1}, \quad \delta_6 = \frac{b}{\alpha_1}, \quad \delta_7 = \frac{\alpha_1}{k_1\omega^2}, \quad \delta_8 = \frac{\alpha_3}{k_2\omega^2},$$

$$\delta_9 = \frac{\gamma_1 T_0}{\alpha_1}, \quad \delta_{10} = \frac{b_1}{c_1^2 k_2}, \quad \delta_{11} = \frac{\gamma}{c_1^2 k_2}, \quad \delta_{12} = \frac{dk_1}{\alpha_1 k_2},$$

$$\delta_{13} = \frac{\alpha_3}{k_2\omega^2}, \quad \delta_{14} = \frac{\alpha_2}{k_2\omega^2}, \quad \delta_{15} = \frac{\gamma_2 T_0 k_1}{a_1 k_2}, \quad \delta_{16} = \frac{k\omega}{\rho C^* c_1^2},$$

$$\delta_{17} = \frac{\beta}{\rho C^*}, \quad \delta_{18} = \frac{\gamma_1 \alpha_1}{\rho C^* k_1 \omega^2}, \quad \delta_{19} = \frac{\gamma_2 \alpha_1}{\rho C^* k_1 \omega^2}, \quad \delta_{20} = \frac{\delta_{16}}{-i\omega} \quad (21)$$

Assuming the time harmonic solution of the equations (17)-(20) as

$$(u(x,t), \varphi(x,t), \psi(x,t), T(x,t)) = (\bar{u}, \bar{\varphi}, \bar{\psi}, \bar{T})e^{-i\omega t} \quad (22)$$

where ω is the frequency

Equations (17)-(20) with the aid of equation (22) yield

$$\bar{u},_{11} = N_1\bar{u} + N_2\bar{\phi},_1 + N_3\bar{\psi},_1 + N_4\bar{T},_1, \quad (23)$$

$$\bar{\phi},_{11} = N_5\bar{u},_1 + N_6\bar{\phi} + N_7\bar{\psi}, N_8\bar{T}, \quad (24)$$

$$\bar{\psi},_{11} = N_9\bar{u},_1 + N_{10}\bar{\phi} + N_{11}\bar{\psi} + N_{12}\bar{T}, \quad (25)$$

$$\bar{T},_{11} = N_{13}\bar{u},_1 + N_{14}\bar{\phi} + N_{15}\bar{\psi} + N_{16}\bar{T}, \quad (26)$$

where

$$N_1 = -\omega^2, \quad N_2 = -\delta_1, \quad N_3 = -\delta_2, \quad N_4 = \delta_3,$$

$$N_5 = \frac{M_1 M_7 + M_2}{M_{15}},$$

$$N_6 = \frac{M_1 M_8 + M_3}{M_{15}}, \qquad N_7 = \frac{M_1 M_9 + M_4}{M_{15}},$$
$$N_8 = \frac{M_1 M_{10} + M_5}{M_{15}}, \qquad N_9 = M_6 N_5 + M_7,$$
$$N_{10} = M_6 N_6 + M_8, \qquad N_{11} = M_6 N_7 + M_9,$$
$$N_{12} = M_6 N_8 + M_{10}.$$

Also

$$M_1 = \frac{-\delta_5}{\delta_4}, \qquad M_2 = \frac{\delta_6}{\delta_4}, \qquad M_3 = \frac{\delta_7 - \omega^2}{\delta_4},$$
$$M_4 = \frac{\delta_8}{\delta_4}, \qquad M_5 = \frac{-\delta_9}{\delta_4}, \qquad M_6 = \frac{-\delta_{10}}{\delta_{11}},$$
$$M_7 = \frac{\delta_{12}}{\delta_{11}}, \qquad M_8 = \frac{\delta_{13}}{\delta_{11}}, \qquad M_9 = \frac{\delta_{14} - \omega^2}{\delta_{11}},$$
$$M_{10} = \frac{-\delta_{15}}{\delta_{11}}, \quad M_{11} = \frac{\delta_{17}}{\delta_{20}}, \quad M_{12} = \frac{\delta_{18}}{\delta_{20}}, \quad M_{13} = \frac{\delta_{19}}{\delta_{20}},$$
$$M_{14} = \frac{1}{\delta_{20}}, \quad M_{15} = 1 - M_1 M_6.$$

State –Space Formulation: Choosing a state variable displacement $\bar{u}$, volume fraction $\bar{\varphi}$ and $\bar{\psi}$, temperature change $\bar{T}$ in the x-direction, the equations can be written in the matrix form as

$$\frac{dV(x,\omega)}{dx} = A(\omega)V(x,\omega), \tag{27}$$

and the values of $A(\omega)$, $V(x,\omega)$ are given in the appendix. The formal solution of system (27) can be written in the form

$$V(x,\omega) = \exp[A(\omega)x]\,V(0,\omega), \tag{28}$$

and the value of $V(0,\omega)$ is given in the appendix.

We shall use the well-known Cayley-Hamilton theorem to find the form of the matrix $\exp[A(\omega)x]$. The characteristic equation of the matrix $A(\omega)$ can be written as

$$\lambda^8 + D_1\lambda^6 + D_2\lambda^4 + D_3\lambda^2 + D_4 = 0, \tag{29}$$

Where

$$D_1 = -N_1 - N_6 - N_{11} - N_{16} - N_2N_5 - N_3N_9 - N_4N_{13},$$

$$\begin{aligned} D_2 = {} & N_1N_6 + N_1N_{11} + N_1N_{16} + N_6N_{11} + N_6N_{16} - N_7N_{10} - N_8N_{14} + \\ & N_{11}N_{16} - N_{12}N_{15} - N_2N_7N_9 + N_3N_6N_9 + N_2N_5N_{11} + N_2N_5N_{16} - \\ & N_3N_5N_{10} + N_3N_9N_{16} - N_2N_8N_{13} - N_4N_5N_{14} + N_4N_6N_{13} - \\ & N_4N_9N_{15} - N_3N_{12}N_{13} + N_4N_{11}N_{13}, \end{aligned}$$

$$\begin{aligned} D_3 = {} & -N_6N_{11}N_{16} + N_7N_{10}N_{16} - N_1N_6N_{11} + N_1N_7N_{10} - N_1N_6N_{16} + \\ & N_1N_8N_{14} - N_1N_{11}N_{16} + N_1N_{12}N_{15} + N_6N_{12}N_{15} - N_7N_{12}N_{14} - \\ & N_8N_{10}N_{15} + N_8N_{11}N_{14} + N_2N_7N_9N_{16} - N_3N_6N_9N_{16} - \\ & N_2N_8N_9N_{15} + N_3N_8N_9N_{14} + N_4N_6N_9N_{15} - N_4N_7N_9N_{14} - \\ & N_2N_5N_{11}N_{16} + N_3N_5N_{10}N_{16} + N_2N_5N_{12}N_{15} - N_2N_7N_{12}N_{13} + \\ & N_2N_8N_{11}N_{13} - N_3N_5N_{12}N_{14} + N_3N_6N_{12}N_{13} - N_3N_8N_{10}N_{13} - \\ & N_4N_5N_{10}N_{15} + N_4N_5N_{11}N_{14} - N_4N_6N_{11}N_{13} + N_4N_7N_{10}N_{13}, \end{aligned}$$

$$\begin{aligned} D_4 = {} & N_1N_6(N_{11}N_{16} - N_{12}N_{15}) + N_1N_7(N_{12}N_{14} - N_{10}N_{16}) + \\ & N_1N_8(N_{10}N_{15} - N_{11}N_{14}) \end{aligned}. \tag{30}$$

Equation (29) is biquartic in λ^2 yield four roots say, λ_1, λ_2, λ_3 and λ_4. Now, the Taylor series expansion for matrix exponential in equation (28) is given by

$$\exp[A(\omega)x] = \sum_{n=0}^{\infty}\left\{\frac{[A(\omega)x]^n}{n!}\right\}. \tag{31}$$

Using Cayley-Hamilton theorem, this infinite series can be truncated as

$$\exp[A(\omega)x] = a_0 I + a_1 A + a_2 A^2 + a_3 A^3 \tag{32}$$

where a_0, a_1, a_2, a_3 are parameters depending on x and ω. According to Cayley-Hamilton theorem the characteristic roots $-\lambda_1$, $-\lambda_2$, $-\lambda_3$, $-\lambda_4$ of the matrix A must satisfy equation (32). Therefore, we get

$$\begin{aligned} \exp[-\lambda_1 x] &= a_0 I - a_1\lambda_1 + a_2\lambda_1^2 - a_3\lambda_1^3, \\ \exp[-\lambda_2 x] &= a_0 I - a_1\lambda_2 + a_2\lambda_2^2 - a_3\lambda_2^3, \\ \exp[-\lambda_3 x] &= a_0 I - a_1\lambda_3 + a_2\lambda_3^2 - a_3\lambda_3^3, \\ \exp[-\lambda_4 x] &= a_0 I - a_1\lambda_4 + a_2\lambda_4^2 - a_3\lambda_4^3. \end{aligned} \tag{33}$$

Solving the above system of equations, we obtain the value of parameters a_0, a_1, a_2, a_3 and these values are given in appendix.

Therefore, we have

$$\exp[A(\omega)x] = L(x,\omega), \tag{34}$$

where $L(x,\omega)$ is a 8x8 matrix with the components

$$l_{11} = a_0 + a_2N_1, \quad l_{12} = a_3R_1, \quad l_{13} = a_3R_2, \quad l_{14} = a_3R_3,$$

$$l_{21} = a_3R_5, \quad l_{22} = a_0 + a_2N_6, \quad l_{23} = a_2N_7, \quad l_{24} = a_2N_8,$$

$$l_{31} = a_3R_9, \quad l_{32} = a_2N_{10}, \quad l_{33} = a_0 + a_2N_{11}, \quad l_{34} = a_2N_{12},$$

$$l_{41} = a_3R_{13}, \quad l_{42} = a_2N_{14}, \quad l_{43} = a_2N_{15}, \quad l_{44} = a_0 + a_2N_{16},$$
(35)

in which

$$R_1 = N_2N_6 + N_3N_{10} + N_4N_{14},$$
$$R_2 = N_2N_7 + N_3N_{11} + N_4N_{15},$$
$$R_3 = N_2N_8 + N_3N_{12} + N_4N_{16},$$
$$R_5 = N_1N_5, R_9 = N_1N_9, \qquad R_{13} = N_1N_{13}.$$

Rewriting the equation (28) with the aid of equation (34) yield,

$$V(x,\omega) = L(x,\omega)V(0,\omega). \tag{36}$$

Therefore, we obtain

$$\begin{bmatrix} \bar{u} \\ \bar{\varphi} \\ \bar{\psi} \\ \bar{T} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{21} & l_{22} & l_{23} & l_{24} \\ l_{31} & l_{32} & l_{33} & l_{34} \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}. \tag{37}$$

Boundary Conditions: A homogenous isotopic thermoelastic solid with double porosity structure occupying the region $0 \le x < \infty$ is considered. The bounding plane $x = 0$ is subjected to a normal force and a thermal source. Mathematically these can be written as,

(i) $t_{11} = -F_1 \exp[-i\omega t],$ (38)

(ii) $\sigma_1 = -F_1 \exp[-i\omega t],$ (39)

(iii) $\tau_1 = -F_1 \exp[-i\omega t],$ (40)

(iv) $T = F_2 \exp[-i\omega t],$ (41)

where F_1 and F_2 are the magnitude of the force and constant temperature applied on the boundary.

Substituting the values of u, φ, ψ, T, t_{11}, σ_1 and τ_1 from equations (8), (9), (10), and (37) in the equations (38)-(41), with the aid of equations (16) and (22), we obtain

$$\begin{bmatrix} Q_1 & Q_2 & Q_3 & Q_4 \\ Q_5 & Q_6 & Q_7 & Q_8 \\ Q_9 & Q_{10} & Q_{11} & Q_{12} \\ Q_{13} & Q_{14} & Q_{15} & Q_{16} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_1 \\ -F_1 \\ F_2 \end{bmatrix}. \tag{42}$$

The values of $Q_1, Q_2, \ldots\ldots\ldots\ldots\ldots, Q_{16}$ are given in the appendix.

Solving (42) for A_1, A_2, A_3, A_4 and substituting the resulting values in equation (37) yield the value of normal stress, equilibrated stress and temperature change.

$$t_{11} = (S_1\frac{\Gamma_1}{\Gamma} + S_2\frac{\Gamma_2}{\Gamma} + S_3\frac{\Gamma_3}{\Gamma} + S_4\frac{\Gamma_4}{\Gamma})e^{-i\omega t}, \tag{43}$$

$$\sigma_1 = (S_5\frac{\Gamma_1}{\Gamma} + S_6\frac{\Gamma_2}{\Gamma} + S_7\frac{\Gamma_3}{\Gamma} + S_8\frac{\Gamma_4}{\Gamma})e^{-i\omega t}, \tag{44}$$

$$\tau_1 = (S_9\frac{\Gamma_1}{\Gamma} + S_{10}\frac{\Gamma_2}{\Gamma} + S_{11}\frac{\Gamma_3}{\Gamma} + S_{12}\frac{\Gamma_4}{\Gamma})e^{-i\omega t}, \tag{45}$$

$$T = (l_{41}\frac{\Gamma_1}{\Gamma} + l_{42}\frac{\Gamma_2}{\Gamma} + l_{43}\frac{\Gamma_3}{\Gamma} + l_{44}\frac{\Gamma_4}{\Gamma})e^{-i\omega t}. \tag{46}$$

Particular Cases

Case I: If $F_2 = 0$ in equations (43)-(46), we obtain the corresponding expression for normal force.

Case II: If $F_1 = 0$ in equations (43)-(46), we get the corresponding expression for thermal source.

NUMARICAL RESULTS AND DISCUSSION

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief and Saleh (39) as

$$\lambda = 7.76\times10^{10}\,Nm^{-2}, \quad C^* = 0.3831\times10^3\,m^2s^{-2}K^{-1},$$
$$\mu = 3.86\times10^{10}\,Nm^{-2},$$
$$k = 3.86\times10^3\,Ns^{-1}K^{-1}, \quad \omega = 1\times10^{11}\,s^{-1},$$
$$T_0 = 0.293\times10^3\,K,$$
$$a = 1.78\times10^{-5}\,K^{-1}, \quad t = 0.1s, \quad \rho = 8.954\times10^3\,Kgm^{-3},$$
$$a_2 = 1.96\times10^{10}\,Nm^{-2}, \quad a_3 = 1.86\times10^{10}\,Nm^{-2},$$
$$\gamma = 0.19\times10^{-5}\,N,$$
$$\gamma_1 = 0.16\times10^5\,Nm^{-2}, \quad b_1 = 0.12\times10^{-5}\,N,$$
$$d = 0.49\times10^{10}\,Nm^{-2},$$
$$\gamma_2 = 0.219\times10^5\,Nm^{-2}K, \quad k_1 = 0.1456\times10^{-12}\,Nm^{-2}s^2,$$
$$b = 0.4\times10^{10}\,Nm^{-2},$$
$$a_1 = 1.65\times10^{10}\,Nm^{-2}, \quad k_2 = 0.1546\times10^{-12}\,Nm^{-2}s^2.$$

The software Matlab has been used to determine the values of normal stress and equilibrated stresses and temperature distribution. The variation of these values with respect to distance *x* are shown in figures (1)-(8), respectively. In all these figures, the curves for double porous viscoelastic medium and double porous thermoelastic medium are represented by VDP and DP respectively.

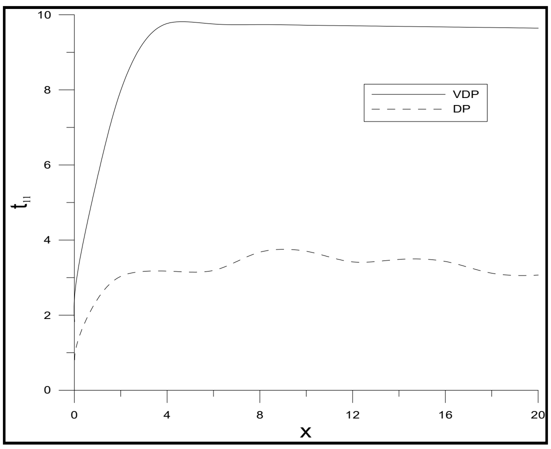

Fig. 1.Variation of normal stress t_{11} with respect to distance x. (Normal force)

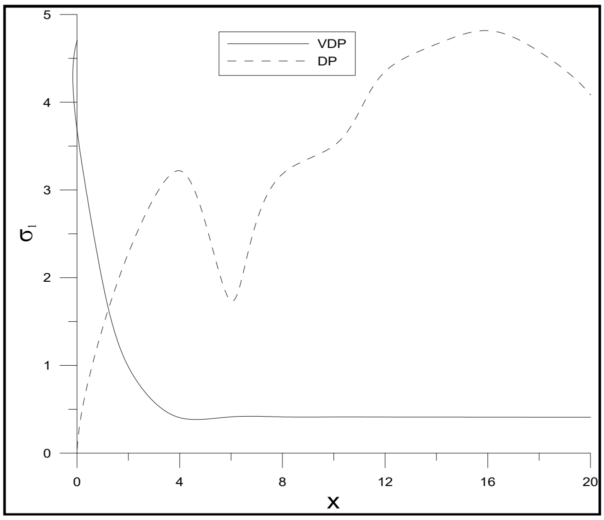

Fig. 2. Variation of equilibrated stress σ_1 with respect to distance x. (Normal force)

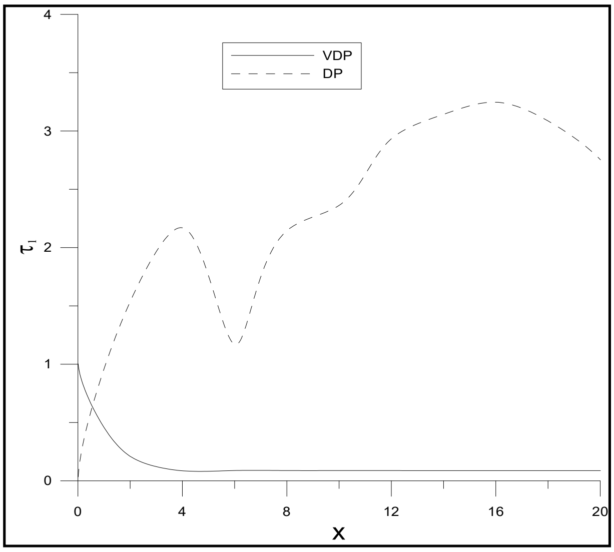

Fig. 3. Variation of equilibrated stress τ_1 with respect to distance x. (Normal force)

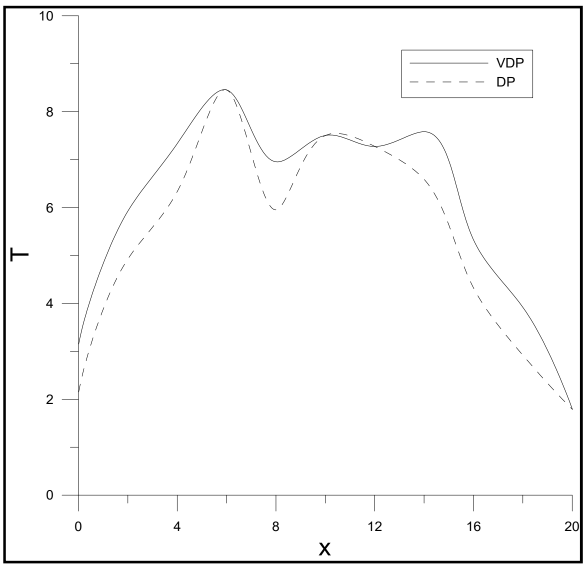

Fig. 4. Variation of temperature distribution *T* with respect to distance *X*. (Normal force)

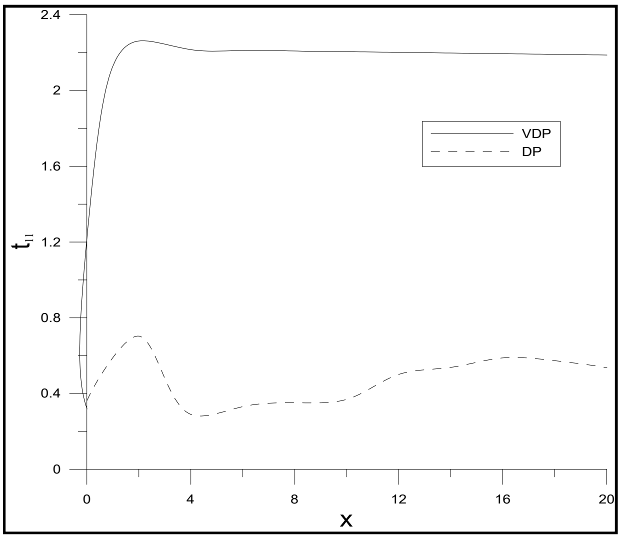

Fig. 5. Variation of normal stress t_{11} with respect to distance x (Thermal source)

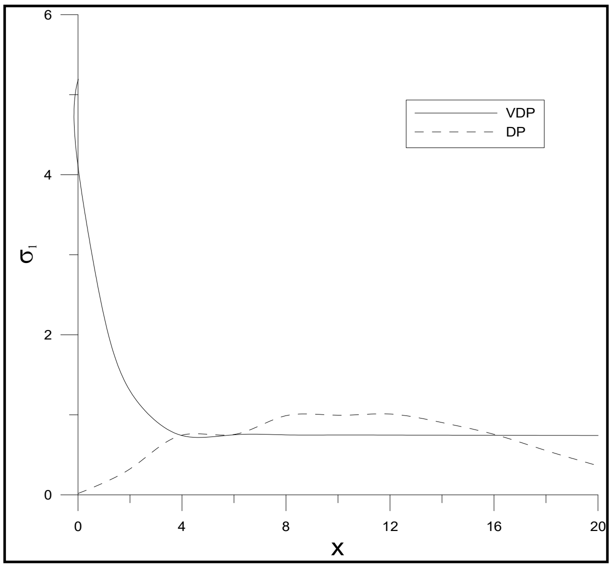

Fig. 6. Variation of equilibrated stress σ_1 with respect to distance x. (Thermal source)

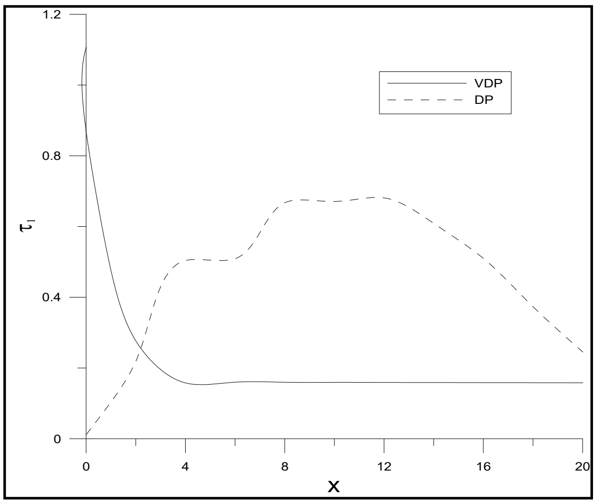

Fig. 7. Variation of equilibrated stress τ_1 with respect to distance x. (Thermal source)

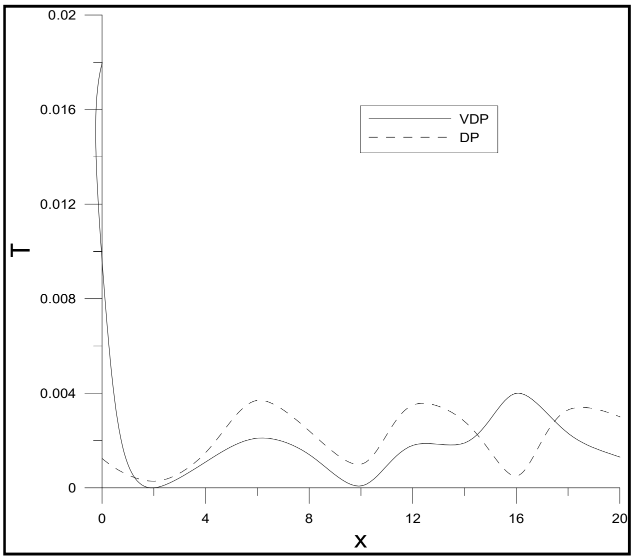

Fig. 8. Variation of temperature distribution T with respect to distance x . (Thermal source)

Normal Force: Fig.1 depicts the variation of normal stress t_{11} with respect to distance x due to normal force. It is found that for VDP, the value of t_{11} increase for the region $0 < x \leq 4$ and then almost constant for the remaining region whereas for DP, it increase for the region $0 < x \leq 2$ and then becomes slightly oscillatory in the remaining region. Also it is noticed that the magnitude value of t_{11} are more for VDP as compared to that DP. mFig.2 and 3 show the variation of equilibrated stresses σ_1 , τ_1 with respect to distance x due to normal force. From figs., it is noticed that for VDP, the values of σ_1 , τ_1 decrease for $0 < x \leq 4$ and then become almost constant for the remaining region, whereas for DP, it shows an oscillatory nature. Also, the magnitude values are higher for DP in comparison to VDP except for the region $0 < x \leq 2$. Fig.4 depicts the variation of temperature distribution T with respect to distance x due to normal force. It is noticed that for both VDP and DP, the value of T increases for the region $0 < x \leq 5$ and then decreases for the remaining region. The trend of variation of T is same for both VDP and DP while the magnitude values of T are higher for VDP than that of DP.

Thermal Source

Fig.5 shows the variation of normal stress t_{11} with respect to distance x due to thermal source. It is found that for VDP, the value of t_{11} increase for the region $0 < x \leq 4$ and then almost constant for the remaining region whereas for DP, it increases for the region $0 < x \leq 2$ decreases for the region $2 < x \leq 4$ and then increases slowly in the remaining region. Also it is noticed that with the magnitude values of t_{11} are more for VDP as compared to that of DP. Fig.6 and 7 depict the variation of equilibrated stresses σ_1 , τ_1 with respect to distance x due to thermal source. From figs., it is noticed that for VDP, the value of σ_1 , τ_1 decrease for $0 < x \leq 4$and then become almost constant for the remaining region whereas for DP, it increases for the region $0 < x \leq 4$ and then decreases for the remaining region. Also, the magnitude values are higher for DP in comparison to VDP except for the region $0 < x \leq 3$.

Fig.8 shows the variation of temperature distribution T with respect to distance $x$due to thermalsource. It is noticed that for both VDP, and DP, the value of T decreases for the region $0 < x \leq 2$and then becomes oscillatory for the remaining region. The trend of variation of T is same for both VDP and DP while the magnitude values of T are higher for DP that that of VDP.

Conclusion

The behavior of normal stress and equilibrated stresses and temperature distribution in an isotropic homogeneous double porous viscoelastic medium has been investigated due to normal force and thermal source. A state space approach has been applied to investigate the problem. The expressions for the components of normal stress, equilibrated stress and the temperature change are obtained in the frequency domain and computed numerically. Numerical simulation is prepared for these quantities and simulated results for these quantities are depicted graphically to show the effect of viscosity. It is observed that viscosity has a significant effect on normal stress, equilibrated stresses and temperature distribution. This type of study is useful due to its application in geophysics and rock mechanics.

APPENDIX

$$A(\omega)=\begin{bmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ N_1&0&0&0&0&N_2&N_3&N_4\\ 0&N_6&N_7&N_8&N_5&0&0&0\\ 0&N_{10}&N_{11}&N_{12}&N_9&0&0&0\\ 0&N_{14}&N_{15}&N_{16}&N_{13}&0&0&0 \end{bmatrix},$$

$$V(x,\omega)=\begin{bmatrix}\bar{u}(x,\omega)\\ \bar{\varphi}(x,\omega)\\ \bar{\psi}(x,\omega)\\ \bar{T}(x,w)\\ (\bar{u}(x,\omega)),_1\\ (\bar{\varphi}(x,\omega)),_1\\ (\bar{\psi}(x,\omega)),_1\\ \bar{T}(x,w)),1\end{bmatrix},\quad V(0,w)=\begin{bmatrix}\bar{u}(0,\omega)\\ \bar{\varphi}(0,\omega)\\ \bar{\psi}(0,\omega)\\ \bar{T}(0,\omega)\\ (\bar{u}(0,\omega)),_1\\ (\bar{\varphi}(0,\omega)),_1\\ (\bar{\psi}(0,\omega)),_1\\ (\bar{T}(0,\omega)),_1\end{bmatrix}.$$

$$a_0=e^{-\lambda_1 x}[1-\frac{\lambda_1\lambda_2(\lambda_3+\lambda_4)+\lambda_1\lambda_3\lambda_4}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}+\frac{\lambda_1{}^2(\lambda_2+\lambda_3+\lambda_4)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}-$$
$$\frac{\lambda_1{}^3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}]-e^{-\lambda_2 x}[\frac{\lambda_1{}^2(\lambda_3+\lambda_4)+\lambda_1\lambda_3\lambda_4}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}-$$
$$\frac{\lambda_1{}^2(\lambda_1+\lambda_3+\lambda_4)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}+\frac{\lambda_1{}^3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}]-$$
$$e^{-\lambda_3 x}[\frac{\lambda_1{}^2(\lambda_2+\lambda_4)+\lambda_1\lambda_2\lambda_4}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)}-\frac{\lambda_1{}^2(\lambda_1+\lambda_2+\lambda_4)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)}+$$
$$\frac{\lambda_1{}^3}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)}]-e^{-\lambda_4 x}[\frac{\lambda_1{}^2(\lambda_2+\lambda_3)+\lambda_1\lambda_2\lambda_3}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}-$$
$$\frac{\lambda_1{}^2(\lambda_1+\lambda_2+\lambda_3)}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}+\frac{\lambda_1{}^3}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}],$$

$$a_1 = -e^{-\lambda_1 x}[\frac{\lambda_2(\lambda_3+\lambda_4)+\lambda_3\lambda_4}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}] - e^{-\lambda_2 x}[\frac{\lambda_1(\lambda_3+\lambda_4)+\lambda_3\lambda_4}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}]$$
$$-e^{-\lambda_3 x}[\frac{\lambda_1(\lambda_2+\lambda_4)+\lambda_2\lambda_4}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)}] - e^{-\lambda_4 x}[\frac{\lambda_1(\lambda_2+\lambda_3)+\lambda_2\lambda_3}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}],$$
$$a_2 = -e^{-\lambda_1 x}[\frac{(\lambda_2+\lambda_3+\lambda_4)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}] - e^{-\lambda_2 x}[\frac{(\lambda_1+\lambda_3+\lambda_4)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}$$
$$-e^{-\lambda_3 x}[\frac{(\lambda_1+\lambda_2+\lambda_4)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)} - e^{-\lambda_4 x}[\frac{(\lambda_1+\lambda_2+\lambda_3)}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)},$$
$$a_3 = -e^{-\lambda_1 x}[\frac{1}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)}] - e^{-\lambda_2 x}[\frac{1}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}]$$
$$-e^{-\lambda_3 x}[\frac{1}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)} - e^{-\lambda_4 x}[\frac{1}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}].$$

$$\mathrm{Q}_1 = P_1(Z_1+N_1Z_3)+P_2(a_3^0R_5)+P_3(a_3^0R_9)-a_3^0R_{13},$$

$$\mathrm{Q}_2 = P_1R_1Z_4+P_2(a_0^0+a_2^0N_6)+P_3(a_0^0+a_2^0N_{11})-a_2^0N_{14},$$

$$\mathrm{Q}_3 = P_1R_2Z_4+P_2(a_2^0N_7)+P_3(a_0^0+a_2^0N_{11})-a_2^0N_{15},$$

$$\mathrm{Q}_4 = P_1R_3Z_4+P_2(a_2^0N_8)+P_3(a_2^0N_{12})-(a_0^0+a_2^0N_{16}),$$

$$\mathrm{Q}_5 = P_4R_5Z_4+P_5R_9Z_4, \qquad \mathrm{Q}_6 = P_4(Z_1+N_6Z_3)+P_5N_{10}Z_3,$$

$$\mathrm{Q}_7 = P_4N_7Z_3+P_5(Z_1+N_{11}Z_3), \qquad \mathrm{Q}_8 = P_4N_8Z_3+P_5N_{12}Z_3,$$

$$\mathrm{Q}_9 = P_5R_5Z_4+P_6R_9Z_4, \qquad \mathrm{Q}_{10} = P_5(Z_1+N_6Z_3)+P_6N_{10}Z_3,$$

$$\mathrm{Q}_{11} = P_5N_7Z_3+P_6(Z_1+N_{11}Z_3), \qquad \mathrm{Q}_{12} = P_5N_8Z_3+P_6N_{12}Z_3,$$

$$\mathrm{Q}_{13} = a_3^0R_{13}, \mathrm{Q}_{14} = a_2^0N_{14}, \qquad \mathrm{Q}_{15} = a_2^0N_{15}, \qquad \mathrm{Q}_{16} = a_0^0+a_2^0N_{16}.$$

where

$$P_1 = \frac{\lambda+2\mu}{\beta T_0}, \quad P_2 = \frac{b\alpha_1}{k_1\omega^2\beta T_0}, \quad P_3 = \frac{d\alpha_1}{k_1\omega^2\beta T_0},$$

$$P_4 = \frac{\alpha_1}{k_1\omega^2}, \quad P_5 = \frac{b_1\alpha_1}{\alpha k_1\omega^2}, \quad P_6 = \frac{\gamma\alpha_1}{\alpha k_1\omega^2}.$$
$$Z_1 = -\lambda_1 D_{11} - \lambda_2 D_{12} - \lambda_3 D_{13} - \lambda_4 D_{14},$$

$$Z_2 = -\lambda_1 D_{21} - \lambda_2 D_{22} - \lambda_3 D_{23} - \lambda_4 D_{24},$$
$$Z_3 = -\lambda_1 D_{31} - \lambda_2 D_{32} - \lambda_3 D_{33} - \lambda_4 D_{34},$$

$$Z_4 = -\lambda_1 D_{41} - \lambda_2 D_{42} - \lambda_3 D_{43} - \lambda_4 D_{44}.$$
$$\mathrm{Y}_1 = -\lambda_1 D_{11}e^{-\lambda_1 x} - \lambda_2 D_{12}e^{-\lambda_2 x} - \lambda_3 D_{13}e^{-\lambda_3 x} - \lambda_4 D_{14}e^{-\lambda_4 x},$$
$$\mathrm{Y}_2 = -\lambda_1 D_{11}e^{-\lambda_1 x} - \lambda_2 D_{12}e^{-\lambda_2 x} - \lambda_3 D_{13}e^{-\lambda_3 x} - \lambda_4 D_{14}e^{-\lambda_4 x},$$
$$\mathrm{Y}_3 = -\lambda_1 D_{21}e^{-\lambda_1 x} - \lambda_2 D_{22}e^{-\lambda_2 x} - \lambda_3 D_{23}e^{-\lambda_3 x} - \lambda_4 D_{24}e^{-\lambda_4 x},$$
$$\mathrm{Y}_4 = -\lambda_1 D_{31}e^{-\lambda_1 x} - \lambda_2 D_{32}e^{-\lambda_2 x} - \lambda_3 D_{33}e^{-\lambda_3 x} - \lambda_4 D_{34}e^{-\lambda_4 x}.$$
$$S_1 = P_1(Y_1+N_1Y_3)+P_2(l_{21})+P_3l_{31}-l_{41},$$
$$S_2 = P_1(R_1Y_4)+P_2(l_{22})+P_3(l_{32})-l_{42},$$

$$S_3 = P_1(R_2Y_4)+P_2(l_{23})+P_3(l_{33})-l_{43},$$
$$S_4 = P_1R_3Y_4+P_2(l_{24})+P_3(l_{34})-(l_{44}),$$

$$S_5 = P_4R_5Y_4+P_5R_9Y_4,$$

$$S_6 = P_4(Y_4+N_6Y_3)+P_5N_{10}Y_3,$$
$$S_7 = P_4N_7Y_3+P_5(Y_1+N_{11}Y_3),$$

$$S_8 = P_4N_8Y_3+P_5N_{12}Y_3,$$
$$S_9 = P_5R_5Y_4+P_6R_9Y_4,$$

$$S_{10} = P_5(Y_1+N_6Y_3)+P_6N_{10}Y_3,$$
$$S_{11} = P_5N_7Y_3+P_6(Y_1+N_{11}Y_3),$$

$$S_{12} = P_5N_8Y_3+P_6N_{12}Y_3,$$
$$D_{11} = 1 - \frac{\lambda_1\lambda_2(\lambda_3+\lambda_4)+\lambda_1\lambda_3\lambda_4}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)} + \frac{\lambda_1^2(\lambda_2+\lambda_3+\lambda_4)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)} - \frac{\lambda_1^3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)},$$

$$D_{12} = -[\frac{\lambda_1^2(\lambda_3+\lambda_4)+\lambda_1\lambda_3\lambda_4}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)} - \frac{\lambda_1^2(\lambda_1+\lambda_3+\lambda_4)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)} + \frac{\lambda_1^3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}],$$

$$D_{13} = -[\frac{\lambda_1^2(\lambda_2+\lambda_4)+\lambda_1\lambda_2\lambda_4}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)} - \frac{\lambda_1^2(\lambda_1+\lambda+_2\lambda_4)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)} + \frac{\lambda_1^3}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)}],$$

$$D_{14} = -[\frac{\lambda_1^2(\lambda_2+\lambda_3)+\lambda_1\lambda_2\lambda_3}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)} - \frac{\lambda_1^2(\lambda_1+\lambda+_2\lambda_3)}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)} + \frac{\lambda_1^3}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}],$$

$$D_{31} = -\frac{(\lambda_2+\lambda_3+\lambda_4)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda)_4},$$
$$D_{32} = -\frac{(\lambda_1+\lambda_3+\lambda_4)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)},$$
$$D_{33} = -\frac{(\lambda_1+\lambda_2+\lambda_4)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)},$$
$$D_{34} = -\frac{(\lambda_1+\lambda_2+\lambda_3)}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)},$$
$$D_{41} = -\frac{1}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)},$$
$$D_{42} = -\frac{1}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)},$$

$$D_{43} = -\frac{1}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)},$$

$$D_{44} = -\frac{1}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}.$$

$$\Gamma = \begin{bmatrix} Q_1 & Q_2 & Q_3 & Q_4 \\ Q_5 & Q_6 & Q_7 & Q_8 \\ Q_9 & Q_{10} & Q_{11} & Q_{12} \\ Q_{13} & Q_{14} & Q_{15} & Q_{16} \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} -F_1 & Q_2 & Q_3 & Q_4 \\ -F_1 & Q_6 & Q_7 & Q_8 \\ -F_1 & Q_{10} & Q_{11} & Q_{12} \\ F_2 & Q_{14} & Q_{15} & Q_{16} \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} Q_1 & -F_1 & Q_3 & Q_4 \\ Q_5 & -F_1 & Q_7 & Q_8 \\ Q_9 & -F_1 & Q_{11} & Q_{12} \\ Q_{13} & F_2 & Q_{15} & Q_{16} \end{bmatrix}, \quad \Gamma_3 = \begin{bmatrix} Q_1 & Q_2 & -F_1 & Q_4 \\ Q_5 & Q_6 & -F_1 & Q_8 \\ Q_9 & Q_{10} & -F_1 & Q_{12} \\ Q_{13} & Q_{14} & F_2 & Q_{16} \end{bmatrix},$$

$$\Gamma_4 = \begin{bmatrix} Q_1 & Q_2 & Q_3 & -F_1 \\ Q_5 & Q_6 & Q_7 & -F_1 \\ Q_9 & Q_{10} & Q_{11} & -F_1 \\ Q_{13} & Q_{14} & Q_{15} & F_2 \end{bmatrix}.$$

and $a_0^0 = a_0,\ a_2^0 = a_2,\ a_3^0 = a_3,\ at\ x = 0.$

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