

RESEARCH ARTICLE

FORMALISM FOR CROSS SECTION OF CLOSED AND OPEN SHELL NUCLEI BY GRAPHICAL METHOD OF SPIN ALGEBRA

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ABSTRACT

To review the works on the earlier developments and uses of spin algebra, we have highlighted their utilization in finding out the transformation co-efficient and the reaction cross sections of stripping and pick up reaction. The Clebsch Gordon coefficients play an important role in nuclear, quantum, particle and other branches. For transfer reactions of the type  $A+a(b+x) = B(A+x)+b$ , we see that the reactions having spin, angular momentum, total angular momentum, energy, parity conservation have maximum peak cross sectional value compared to the others.

Key words:

Spin algebra in graphical methods,  
Closed shell nuclei,  
CG coefficients,  
Open shell nuclei

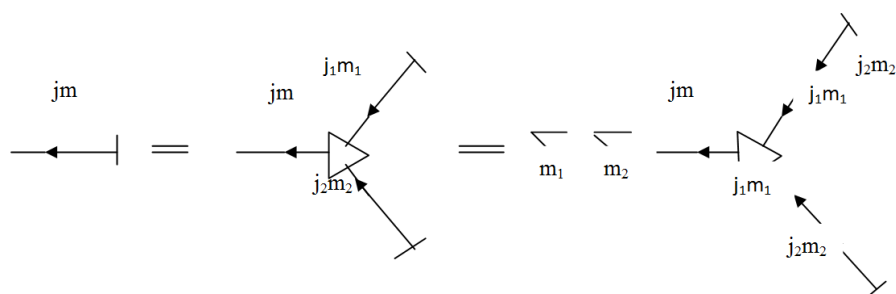
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INTRODUCTION

The graphical methods of spin algebra are done by the Canadian scientists J. Paldus et al for closed as well as open shell nuclei by considering configuration interaction in 1977. In that paper two coupling schemes are considered. When an unpaired electron is added to a closed shell for mono-, bi-excited states maintaining the spin symmetry are discussed [1,2]. The reaction cross sections for the deuteron stripping reactions for the  $s^{1/2}$ ,  $d_{5/2}^{+}$  states for oxygen and calcium are calculated when the coulomb potential in terms of Laguerre polynomial are included [3,4]. The graphical method of spin algebra serve as an useful tool to simplify various complicated and rather formidable expressions into simpler ones for easy computation.

Present works

Graphically the coupling between two angular momentum states  $j_1$  and  $j_2$  is  $|(j_1 j_2)jm\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle |j_1 m_1\rangle |j_2 m_2\rangle$  can be easily represented by spin algebra as shown below



This implies that the coupled wave function  $|(j_1 j_2)jm\rangle$  expressed as a linear combination of uncoupled functions i.e.

$$|(j_1 j_2)jm\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle |j_1 m_1\rangle |j_2 m_2\rangle$$

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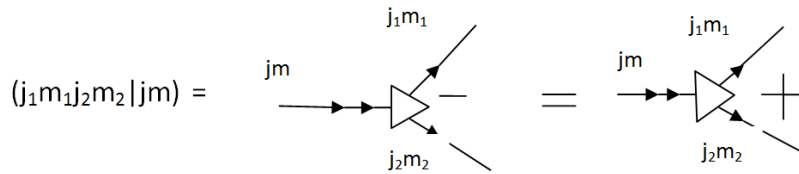
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Further the uncoupled state can be expressed as a combination of coupled states

$$|j_1 m_1\rangle |j_2 m_2\rangle = |(j_1 j_2) j m\rangle \langle j m | j_1 m_1 j_2 m_2\rangle$$

$$M = m_1 + m_2$$

Graphically,



The ‘-’ or ‘+’ sign at the vertex means that the angular momenta have to be read clockwise or anticlockwise. The change of sign at the vertex gives a phase factor  $(-)^{j_1 + j_2 - j}$  where  $j$  is the unique vector.

To introduce a wider symmetry into the properties of vector coupling co-efficient there must be a connection between the CG co-efficient and the Wigner’s 3jm symbol.

Wigner’s 3jm symbol is denoted as

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

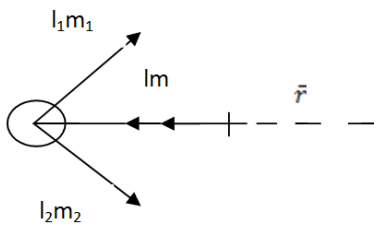
, related to the CG co-efficients as

$$\langle j_1 m_1 j_2 m_2 | j m \rangle = \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} (-)^{j_1 - j_2 + m} [j]$$

Where  $[j] = (2j+1)^{1/2}$

Product of two spherical harmonics in spin algebra is [4]

$$P_{l_1}(\cos \theta) P_{l_2}(\cos \theta) / 16\pi^2 = \frac{1}{(2l_1+1)(2l_2+1)}$$



Considering symmetry and transformation between CG co-efficients and the Wigner’s 3jm symbol for a reaction (i) the arrow in  $j$  is unique in character (ii) one of the arrows which in the CG co-efficients is necessarily  $2^-$  or  $3^+$  introduces a phase  $(-)^{2j}$  (iii) the transformation of the sign at the vertex is changed (iv) the three angular momenta are numbered as 1, 2 and 3 where 1 stands for the unique momentum  $j$  and (v) the variance ket or bra is denoted by superscript ‘+’ or ‘-’.

In order to reduce the multi dimensional AGS coupled integral equation, we have to solve the integral in three different regions by choosing appropriate points and weights for on shell and off shell regions.

$$P = E^{1/2} \frac{1+x}{2}, \quad w_p = \frac{\sqrt{E}}{2} w_x \quad \text{for } 0 \leq x \leq E$$

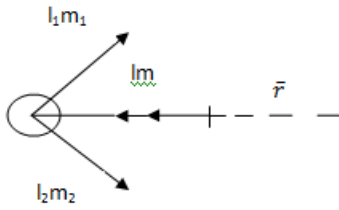
$$P = \left( \frac{2(E+2EB)}{(1-x)} \right)^{1/2}, \quad w_p = \sqrt{\frac{p}{2(1-x)}} w_x \quad \text{for } E \leq x \leq 2EB$$

$$P = E_B(x+1) + E, \quad w_p = \frac{1}{2} \frac{E_B}{(E+2EB)} w_x \quad \text{for } E_B \leq x \leq \infty$$

When the Alt Grassberger & Sandhas(AGS) equation is written in between the bound and free states, the square of the transition amplitude is related to the cross section by the expression

$$\langle Q_i d_i \sum_{l_i s_i} \phi_{(l_i s_i)}^{n_i} I U_{ij}(z) I Q_j d_j \sum_{l_j s_j} \phi_{(l_j s_j)}^{n_j}(z) \rangle = \frac{1}{(2s_j+1)(2J_j+1)} \left( \frac{1}{12} Q_i Q_j \right) (N_i N_j)^2$$

$$\sum_{J,J'} (2J+1)(2J'+1) \sum_{L=(J-1)}^{(J+1)} \sum_{L'=(J'-1)}^{(J'+1)} |T_{ij}|^2 \frac{1}{(2L+1)(2L'+1)}$$



For closed and open shell nuclei the values of  $Q_i$ ,  $Q_j$ ,  $N_i$ ,  $N_j$ ,  $S_j$ ,  $J_j$ ,  $l_j$ ,  $J$  and  $J'$  can be calculated for different levels and hence the reaction cross section.

### Conclusion

The use of the graphical method of spin algebra enables us to obtain the geometrical part of few to many body problem independent of the type of interaction. For this well defined rules consistent with various theorems in angular momentum algebra are formulated to transform complicated diagrammatic expressions into simpler ones. So far we have calculated the reaction cross-sections for the deuteron stripping reactions on  $^{40}\text{Ca}$ ,  $^{16}\text{O}$ ,  $^{28}\text{Si}$ , alpha clustering reactions and some of their stable isotopes by spin algebra for [135X135] matrix. For higher order of matrix and super heavy elements the studies are going on.

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